17301

16172

3 Hours / 100 Marks   Seat No.

Instructions – (1) All Questions are *Compulsory*.

(2) Answer each next main Question on a new page.

(3) Illustrate your answers with neat sketches wherever necessary.

(4) Figures to the right indicate full marks.

(5) Assume suitable data, if necessary.

(6) Use of Non-programmable Electronic Pocket Calculator is permissible.

(7) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1.   **Attempt any TEN of the following:**   **20**

a) Find the point on the curve $y = x^2 - 6x + 8$ where the tangent is parallel to X – axis.

b) Find the radius of curvature of the curve $xy = c$ at point (c, c).

c) Evaluate $\int \dfrac{1}{\sin^{-1}x\sqrt{1-x^2}}\,dx$

d) Evaluate $\int \dfrac{1}{\sqrt[4]{(2-3x)^3}}\,dx$

e) Evaluate $\int \tan^{-1}x\,dx$

f) Evaluate $\displaystyle\int_{0}^{\pi/2} \sin^3 x \, dx$

g) Find the area of the region bounded by $x^2 = 16y, y = 1, y = 4$ and Y-axis in first quadrant.

h) Determine the order and degree of $x^2 \dfrac{d^2 y}{dx^2} + x \dfrac{dy}{dx} = my$

i) Form the differential equation if $y = 4(x - A)^2$. Where A is arbitrary constant.

j) A fair die is rolled. What is the probability that the number on the die is a prime number.

k) From 4 men and 2 women, 3 persons are chosen at random to form a committee. Find the probability that the committee consists of at least one person of either sex.

l) A person fires 10 shorts at target. The probability that any shot will hit the target is $\dfrac{3}{5}$. Find the probability that the target is hit exactly 5 times.

m) Evaluate $\displaystyle\int \dfrac{1}{\sqrt{9 - 4x^2}} dx$

n) Evaluate $\displaystyle\int_{1}^{e} \dfrac{1}{x} . \log x \, dx$

2. **Attempt any FOUR of the following:** **16**

a) Evaluate $\displaystyle\int \tan^8 x \, dx$

b) Evaluate $\displaystyle\int \dfrac{\log x}{x(2 + \log x)(3 + \log x)} dx$

c) Find the equations of the tangent and normal to the ellipse $2x^2 + 3y^2 = 5$ which is perpendicular to the line $3x + 2y + 7 = 0$

d) Find the radius of curvature for the curve $x = a\cos^3 \theta, y = a\sin^3 \theta$ at $\theta = \dfrac{\pi}{4}$.

Marks

e) A bullet is fired into a mud tank and penetrates $(120t - 3600t^2)$ meters in 't' seconds after impact. Calculate maximum depth of penetration.

f) Evaluate $\int \dfrac{5x - 4}{x^2 - 8x + 12} \, dx$

3. **Attempt any FOUR of the following:** 16

a) Evaluate $\displaystyle\int_{-3/2}^{1/2} \dfrac{1}{4x^2 + 12x + 13} \, dx$

b) Evaluate $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \dfrac{1}{1 + \sqrt[3]{\cot x}} \, dx$

c) Find the area of the region enclosed between parabola $y = x^2 + 1$ and the line $y = 2x + 1$.

d) Solve $x^2 \cdot \dfrac{dy}{dx} = x^2 + xy + y^2$

e) Solve $\cos^2(x - 2y) = 1 - 2\dfrac{dy}{dx}$

f) Solve $(1 + x^2)\dfrac{dy}{dx} + y = e^{\tan^{-1}x}$

4. **Attempt any FOUR of the following:** 16

a) Evaluate $\displaystyle\int_{3}^{7} \dfrac{(10 - x)^2}{x^2 + (10 - x)^2} \, dx$

b) Evaluate $\displaystyle\int_{0}^{1} x \cdot \sin^{-1}x \, dx$

c) Find the area of the region in the first quadrant enclosed by the X - axis the line $y = x$ and the circle $x^2 + y^2 = 8$.

d) Solve $y^3 \cdot \sec^2 x \, dx + (3y^2 \tan x - \sec^2 y) \, dy = 0$

e) Solve $(x + y + 1)^2 \dfrac{dy}{dx} = 1$

f) Verify that $y = e^{m \cdot \sin^{-1}x}$ is a solution of differential equation.
$(1 - x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - m^2y = 0$

5.   **Attempt any FOUR of the following:**   **16**

a)   A card is drawn at random from a well shuffled pack of 52 playing cards.
A = event that the card drawn is not a spade.
B = event that the card drawn is king.
verify that events A and B are independent.

b)   Assuming that the probability of a fatal accident during the year is $\dfrac{1}{1200}$. Calculate the probability that in a factory employing 300 workers there will be at least two fatal accidents in a year. $\left[e^{-0.25} = 0.7788\right]$

c)   In a certain examination 500 students appeared. Mean score is 68 with S.D 8. Find the number of students scoring

(i)   less than 50

(ii)   more than 60

d)   Evaluate $\displaystyle\int \dfrac{1}{4 + 5\sin(2x)}\,dx$

e)   Evaluate $\displaystyle\int_{0}^{\frac{\pi}{2}} \dfrac{\sin(2x)}{4 - \sin^2 x}\,dx$

f)   Solve $(2x + e^x \cdot \log y)\,dx + \left(\dfrac{e^x}{y} + 1\right)dy = 0$

Marks

6. **Attempt any <u>FOUR</u> of the following:** 16

a) A husband and wife appeared in an interview for two vacancies in an office. The probability of husband's selection is $\frac{1}{7}$ and that of wife's selection is $\frac{1}{5}$. Find the probability that

   (i) both of them are selected.

   (ii) only one of them is selected.

b) A company manufactures electric motors. The probability that an electric motor is defective is 0.01. What is the probability that a sample of 300 electric motors will contain exactly 5 defective motors? $[e^{-3} = 0.0498]$

c) In a test of 2000 electric bulbs it was found that the life of particular make was normally distributed with average life of 2040 hrs. and S.D. of 60 hrs. Estimate the number of bulbs likely to burn for

   (i) between 1920 hrs and 2160 hrs.

   (ii) more than 2150 hrs.

   Given that A(2) = 0.4772

                  A(1.83) = 0.4664

d) Find the maximum and minimum values of $2x^3 - 3x^2 - 36x + 10$

e) Find the equations of the tangent and the normal to the curve $2x^2 - xy + 3y^2 = 18$ at (3, 1)

f) Find by integration the area of the ellipse $4x^2 + 9y^2 = 36$

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