WINTER– 16 EXAMINATION
Model Answer

Subject Code: **17301**

Important Instructions to examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills.
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q. N.	Answer	Marking Scheme
1		**Attempt any TEN of the following:**	**20**
	a)	Find 'a' if the tangent to the curve $y = x^2 + ax$ at the origin is parallel to the line passing through $A(-4,-3)$ and $B(-2,5)$	**02**
	Ans	$y = x^2 + ax$	
		$\therefore \frac{dy}{dx} = 2x + a$	½
		at origin	
		$\frac{dy}{dx} = a$	½
		$\therefore$ slope of tangent $= a$	
		slope of line AB is, $m = \frac{y_2 - y_1}{x_2 - x_1}$	
		$\therefore m = \frac{5+3}{-2+4} = 4$	½
		$\because$ tangent is parallel to AB	
		$\therefore a = 4$	½
		--	
	b)	Find Radius of curvature of $y = x^3$ at $(1,1)$.	**02**
	Ans	$y = x^3$	
		$\because \frac{dy}{dx} = 3x^2$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
1	b)	$\frac{d^2y}{dx^2} = 6x$ at $(1,1)$ $\frac{dy}{dx} = 3$ $\frac{d^2y}{dx^2} = 6$ Radius of curvature $\rho = \frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$ $\therefore\ \rho == \frac{\left(1+3^2\right)^{\frac{3}{2}}}{6} = 5.27$	½ 1
	c) Ans	Evaluate: $\int\left(e^x + x^e + e^e\right)dx$ $\int\left(e^x + x^e + e^e\right)dx$ $= e^x + \frac{x^{e+1}}{e+1} + e^e x + c$	**02** 2
	d) Ans	Evaluate: $\int \frac{1}{x+\sqrt{x}}dx$ $\int \frac{1}{x+\sqrt{x}}dx$ $\therefore\ I = \int \frac{1}{\sqrt{x}\sqrt{x}+\sqrt{x}}dx$ $I = \int \frac{1}{\sqrt{x}\left(\sqrt{x}+1\right)}dx$ put $\sqrt{x}+1 = t$ $\therefore\ \frac{1}{\sqrt{x}}dx = 2dt$ $\therefore\ I = \int \frac{2dt}{t}$ $I = 2\log t + c$ $I = 2\log\sqrt{x} + c$	**02** ½ ½ ½ ½

Q. No.	Sub Q. N.	Answer	Marking Scheme
1	e)	Evaluate $\int \sin^2 x\, dx$	**02**
	Ans	$\int \sin^2 x dx$	
		$\therefore I = \int \frac{1-\cos 2x}{2} dx$	1
		$I = \frac{1}{2}\left(x - \frac{\sin 2x}{2}\right) + c$	1
	f)	Evaluate: $\int \sec^2 x.x.dx$	**02**
	Ans	$\int \sec^2 x.x.dx$	
		$= x\int \sec^2 xdx - \int\left(\int \sec^2 xdx.\frac{d}{dx}x\right)dx$	½
		$= x\tan x - \int \tan x.1dx$	1
		$= x\tan x - \log(\sec x)dx + c$	½
	g)	Find 'k',if $\int_0^1 (3x^2 + 2x + k)dx = 0$	**02**
	Ans	$\int_0^1 (3x^2 + 2x + k)dx = 0$	
		$\therefore \left[3\frac{x^3}{3} + 2\frac{x^2}{2} + kx\right]_0^1 = 0$	1
		$\therefore 1 + 1 + k = 0$	½
		$\therefore k = -2$	½
	h)	Evaluate: $\int \cos ec^2 (x^0)dx$	**02**
	Ans	$\int \cos ec^2 (x^0)dx$	
		$\therefore I = \int \cos ec^2\left(\frac{\pi x}{180}\right)dx$	1
		$\therefore I = -\frac{\cot\left(\frac{\pi x}{180}\right)}{\frac{\pi}{180}} + c$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
1	h)	$\therefore I = -\cot\left(\frac{\pi x}{180}\right)\frac{180}{\pi} + c$	
	i)	Find the area under the parabola $y^2 = 4x$ bounded by the lines $x = 0, y = 0, x = 4$	**02**
	Ans	$y^2 = 4x$ $x = 0, y = 0, x = 4$ $y = 2\sqrt{x}$	
		Area $A = \int_0^4 2\sqrt{x}\, dx$	½
		$\therefore A = 2\int_0^4 x^{\frac{1}{2}} dx$	
		$\therefore A = 2\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4$	½
		$\therefore A = \frac{4}{3}\left[4^{\frac{3}{2}}\right]$	½
		$\therefore A = \frac{32}{3}$ or 10.67	½
	j)	Find order and degree of the differntial equation $\frac{d^2y}{dx^2} = \left(y + \frac{dy}{dx}\right)^{\frac{3}{2}}$	**02**
	Ans	$\frac{d^2y}{dx^2} = \left(y + \frac{dy}{dx}\right)^{\frac{3}{2}}$ Order = 2	1
		$\therefore \left(\frac{d^2y}{dx^2}\right)^2 = \left(y + \frac{dy}{dx}\right)^3$ Degree = 2	1
	k)	Form a D.E. if $y = A\sin x + B\cos x$	**02**
	Ans	$y = A\sin x + B\cos x$	
		$\therefore \frac{dy}{dx} = A\cos x - B\sin x$	½

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1	k)	$\therefore \frac{d^2y}{dx^2} = -A\sin x - B\cos x$	½
		$\therefore \frac{d^2y}{dx^2} = -(A\sin x + B\cos x)$	½
		$\therefore \frac{d^2y}{dx^2} = -y$	½
		$\therefore \frac{d^2y}{dx^2} + y = 0$	
		--	
	l)	Form a differential equation, if $y = ax^2 + b$	**02**
	Ans	$y = ax^2 + b$	
		$\therefore \frac{dy}{dx} = 2ax$	½
		$\therefore \frac{d^2y}{dx^2} = 2a$	½
		$\therefore \frac{d^2y}{dx^2} = \frac{1}{x}\frac{dy}{dx}$	1
		$\therefore x\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$	
		--	
	m)	Find the probability of getting sum of numbers is 9 with two dice.	**02**
	Ans	$S = \{(1,1)(1,2)(1,3)(1,4)(1,5)(1,6)$	
		$(2,1)(2,2)(2,3)(2,4)(2,5)(2,6)$	
		$(3,1)(3,2)(3,3)(3,4)(3,5)(3,6)$	
		$(4,1)(4,2)(4,3)(4,4)(4,5)(4,6)$	
		$(5,1)(5,2)(5,3)(5,4)(5,5)(5,6)$	
		$(6,1)(6,2)(6,3)(6,4)(6,5)(6,6)\}$	
		$n(s) = 36$	½
		sum of numbers is 9	
		$\therefore A = \{(4,5)\ (5,4)\ (3,6)\ (6,3)\}$	
		$\therefore n(A) = 4$	½
		$p(A) = \frac{n(A)}{n(s)}$	
		$p(A) = \frac{4}{36}$ or 0.111	1

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1	n)	An unbaised coin is tossed 3 times . Find the probability of getting two head	**02**
	Ans	$S = \{HHH, HTT, THT, TTH, HTH, HHT, THH, TTT\}$	
		$\therefore n(S) = 8$	½
		$A = \{HHT, HTH, THH\}$	
		$n(A) = 3$	½
		$\therefore p(A) = \dfrac{n(A)}{n(S)}$	
		$\therefore p(A) = \dfrac{3}{8}$ or 0.375	1
		--	
2		**Attempt any Four of the following:**	**16**
	a)	Find the equation of tangent to the circle $x^2 + y^2 + 6x - 6y - 7 = 0$ at point it cuts the x – axis.	**04**
	Ans	$x^2 + y^2 + 6x - 6y - 7 = 0$	
		curve cuts the x – axis $\therefore y = 0$	
		$\therefore x^2 + 6x - 7 = 0$	
		$\therefore x = 1, -7$	½
		points are $(1,0)$ and $(-7,0)$	
		$x^2 + y^2 + 6x - 6y - 7 = 0$	
		$\therefore 2x + 2y\dfrac{dy}{dx} + 6 - 6\dfrac{dy}{dx} = 0$	
		$\therefore (2y - 6)\dfrac{dy}{dx} = -2x - 6$	
		$\therefore \dfrac{dy}{dx} = \dfrac{-2x-6}{2y-6}$	½
		at $(1,0)$	
		$\therefore \dfrac{dy}{dx} = \dfrac{-2-6}{0-6} = \dfrac{4}{3}$	½
		$\therefore$ slope $m = \dfrac{4}{3}$	
		equation of tangent is	
		$(y - 0) = \dfrac{4}{3}(x - 1)$	
		$\therefore 3y = 4x - 4$	1
		$\therefore 4x - 3y - 4 = 0$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
2	a)	at $(-7,0)$	
		$\therefore \dfrac{dy}{dx}=\dfrac{-2(-7)-6}{0-6}=\dfrac{-4}{3}$	½
		$\therefore$ slope $m=\dfrac{-4}{3}$	
		equation of tangent is	
		$(y-0)=\dfrac{-4}{3}(x+7)$	
		$\therefore 3y=-4x-28$	
		$\therefore 4x+3y+28=0$	1
		--	
	b)	Discuss the maxima and minima of the function "$\tan x-2x$"	**04**
	Ans	Let $y=\tan x-2x$	
		$\therefore \dfrac{dy}{dx}=\sec^2 x-2$	½
		$\therefore \dfrac{d^2y}{dx^2}=2\sec x\sec x\tan x=2\sec^2 x\tan x$	½
		consider $\dfrac{dy}{dx}=0$	
		$\therefore \sec^2 x-2=0$	
		$\therefore \sec^2 x=2$ or $\tan^2 x-1=0$ $\therefore \tan x=1$ or $\tan x=-1$	
		$\therefore \sec x=\sqrt{2},-\sqrt{2}$ or $\cos x=\dfrac{1}{\sqrt{2}},-\dfrac{1}{\sqrt{2}}$	
		$\therefore x=\dfrac{\pi}{4},\dfrac{3\pi}{4}$ or $x=\dfrac{\pi}{4},\dfrac{3\pi}{4}$ or $x=\dfrac{\pi}{4},-\dfrac{\pi}{4}$	1
		at $x=\dfrac{\pi}{4}$	
		$\therefore \dfrac{d^2y}{dx^2}=2\sec^2\left(\dfrac{\pi}{4}\right)\tan\left(\dfrac{\pi}{4}\right)=2(2)(1)=4>0$	½
		$\therefore$ function is minimum at $x=\dfrac{\pi}{4}$	
		$\therefore y_{min}=\tan\dfrac{\pi}{4}-\dfrac{\pi}{2}=1-\dfrac{\pi}{2}$	½
		at $x=\dfrac{3\pi}{4}$ or $x=-\dfrac{\pi}{4}$	
		$\therefore \dfrac{d^2y}{dx^2}=2\sec^2\left(\dfrac{3\pi}{4}\right)\tan\left(\dfrac{3\pi}{4}\right)=-4<0$	½

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2	b)	$\therefore$ function is maximum at $x = \frac{3\pi}{4}$	
		$\therefore y_{max} = \tan\left(\frac{3\pi}{4}\right) - \frac{3\pi}{2} = -1 - \frac{3\pi}{2}$ or $-1 - \frac{\pi}{2}$	½
		--	
	c)	Find the radius of curvature of the curve $\sqrt{x} + \sqrt{y} = 1$ at $\left(\frac{1}{4}, \frac{1}{4}\right)$	**04**
	Ans	$\sqrt{x} + \sqrt{y} = 1$	
		$\therefore \frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0$	½
		$\therefore \frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$	
		$\frac{d^2y}{dx^2} = \frac{-\left[\sqrt{x}\frac{1}{2\sqrt{y}}\frac{dy}{dx} - \sqrt{y}\frac{1}{2\sqrt{x}}\right]}{\left(\sqrt{x}\right)^2}$	½
		$\frac{d^2y}{dx^2} = \frac{-\left[\sqrt{x}\frac{1}{2\sqrt{y}}\left(-\frac{\sqrt{y}}{\sqrt{x}}\right) - \sqrt{y}\frac{1}{2\sqrt{x}}\right]}{x}$	
		$\frac{d^2y}{dx^2} = \frac{-\left[-\frac{1}{2} - \frac{\sqrt{y}}{2\sqrt{x}}\right]}{x}$	
		$\therefore$ at $\left(\frac{1}{4}, \frac{1}{4}\right)$	
		$\frac{dy}{dx} = -\frac{\sqrt{\frac{1}{4}}}{\sqrt{\frac{1}{4}}} = -1$	1
		$\frac{d^2y}{dx^2} = \frac{-\left[-\frac{1}{2} - \frac{1}{2}\right]}{\frac{1}{4}} = 4$	1
		$\therefore$ Radius of curvature is $\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$	

<table>
<tr><th>Q. No.</th><th>Sub Q. N.</th><th>Answer</th><th>Marking Scheme</th></tr>
<tr><td>2</td><td>c)</td><td>$\therefore \rho = \frac{\left[1+(-1)^2\right]^{\frac{3}{2}}}{4}$
$\therefore \rho = 0.707$</td><td>½
½</td></tr>
<tr><td></td><td>d)
Ans</td><td>Evaluate: $\int \tan^6 x \, dx$
$\int \tan^6 x \, dx$
$= \int \tan^4 x \tan^2 x \, dx$
$= \int \tan^4 x \left(\sec^2 x - 1\right) dx$
$= \int \tan^4 x \sec^2 x \, dx - \int \tan^4 x \, dx$
$= \int \tan^4 x \sec^2 x \, dx - \int \tan^2 x \tan^2 x \, dx$
$= \int \tan^4 x \sec^2 x \, dx - \int \tan^2 x \left(\sec^2 x - 1\right) dx$
$= \int \tan^4 x \sec^2 x \, dx - \left[\int \tan^2 x \sec^2 x \, dx - \int \tan^2 x \, dx\right]$
$= \int \tan^4 x \sec^2 x \, dx - \left[\int \tan^2 x \sec^2 x \, dx - \int \left(\sec^2 x - 1\right) dx\right]$
$= \int \tan^4 x \sec^2 x \, dx - \left[\int \tan^2 x \sec^2 x \, dx - \left(\tan x - x\right)\right] + c$
put $\tan x = t$
$\therefore \sec^2 x dx = dt$
$\therefore I = \int t^4 dt - \left[\int t^2 dt - I_1\right] + c \qquad \because I_1 = \tan x - x$
$= \frac{t^5}{5} - \left[\frac{t^5}{5} - I_1\right] + c$
$= \frac{\tan^5 x}{5} - \left[\frac{\tan^3 x}{3} - I_1\right] + c$
$= \frac{\tan^5 x}{5} - \frac{\tan^3 x}{3} + \tan x - x + c$</td><td>04
½
½
½
½
½
½
½
½</td></tr>
<tr><td></td><td>e)
Ans</td><td>Evaluate: $\int \cos\left(\log x\right) dx$
$\int \cos\left(\log x\right) dx$
Put $\log x = t \Rightarrow x = e^t$
$\therefore \frac{1}{x} dx = dt$
$\therefore dx = x dt$
$\therefore dx = e^t dt$</td><td>04
½</td></tr>
</table>

Q. No.	Sub Q. N.	Answer	Marking Scheme
2	e)	$\therefore\ I=\int e^{t}\cos t\,dt$	½
		$=\dfrac{e^{t}}{1+1}\left(1\cos t+1\sin t\right)+c$	2
		$=\dfrac{x}{2}\left(\cos\left(\log x\right)+\sin\left(\log x\right)\right)+c$	1
		OR	
		$\int\cos\left(\log x\right)\ dx$	
		$\text{Put}\log x=t\Rightarrow\ x=e^{t}$	
		$\therefore\ \dfrac{1}{x}dx=dt$	
		$\therefore\ dx=xdt$	½
		$\therefore\ dx=e^{t}dt$	½
		$\therefore\ I=\int e^{t}\cos t\,dt$	½
		$=\cos t\int e^{t}dt-\int\left(\int e^{t}dt\dfrac{d}{dx}x\right)dx$	½
		$=\cos t\ e^{t}-\int e^{t}\left(-\sin t\right)dt$	
		$=\cos t\ e^{t}+\int e^{t}\sin t\,dt+c$	½
		$=\cos t\ e^{t}+e^{t}\sin t-\int e^{t}\cos t\,dt+c$	½
		$\therefore\ I=\cos t\ e^{t}+e^{t}\sin t-I+c$	
		$\therefore\ 2I=\cos t\ e^{t}+e^{t}\sin t+c$	
		$\therefore\ I=\dfrac{e^{t}}{2}\left(\cos t\ +\sin t\right)+c$	½
		$\therefore\ I=\dfrac{x}{2}\left(\cos\left(\log x\right)+\sin\left(\log x\right)\right)+c$	½
		OR	
		$I=\int\cos\left(\log x\right)\ dx$	½
		$\therefore\ I=\int\cos\left(\log x\right).1\ dx$	
		$\therefore\ I=\cos\left(\log x\right)\int 1dx-\int\left(\int 1dx\dfrac{d}{dx}\cos\left(\log x\right)\right)dx$	½
		$\therefore\ I=\cos\left(\log x\right)x-\int x\left(\dfrac{-\sin\left(\log x\right)}{x}\right)dx$	1
		$\therefore\ I=x\cos\left(\log x\right)+\int\sin\left(\log x\right)\ dx$	

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2		$\therefore\ I = x\cos(\log x) + \int \sin(\log x).1dx$	½
		$\therefore\ I = x\cos(\log x) + \sin(\log x)x - \int x\left(\frac{\cos(\log x)}{x}\right)dx$	½
		$\therefore\ I = x\cos(\log x) + x\sin(\log x) - \int \cos(\log x)\ dx$	
		$\therefore\ I = x\cos(\log x) + x\sin(\log x) - I$	½
		$\therefore\ 2I = x\left(\cos(\log x) + \sin(\log x)\right)$	
		$\therefore\ I = \frac{x}{2}\left(\cos(\log x)x + x\sin(\log x)\right) + c$	½
		--	
	f)	Evaluate: $\int \frac{\sec^2 x}{(1+\tan x)(2+\tan x)}dx$	**04**
	Ans	$\int \frac{\sec^2 x}{(1+\tan x)(2+\tan x)}dx$	
		Put $\tan x = t$	1
		$\sec^2 x dx = dt$	
		$\therefore\ I = \int \frac{dt}{(1+t)(2+t)}$	
		Let $\frac{1}{(1+t)(2+t)} = \frac{A}{1+t} + \frac{B}{2+t}$	½
		$1 = A(2+t) + B(1+t)$	
		Put $t = -1$	
		$1 = A(1)$	½
		$\therefore\ A = 1$	
		Put $t = -2$	
		$1 = B(-1)$	½
		$\therefore\ B = -1$	
		$\therefore\ \frac{1}{(1+t)(2+t)} = \frac{1}{1+t} + \frac{-1}{2+t}$	
		$\therefore\ \int \frac{dt}{(1+t)(t+2)} = \int\left(\frac{1}{1+t} + \frac{-1}{2+t}\right)dt$	½ ½
		$\therefore\ I = \log(1+t) - \log(2+t) + c$	
		$\therefore\ I = \log(1+\tan x) - \log(2+\tan x) + c$ or $I = \log\left(\frac{1+\tan x}{2+\tan x}\right) + c$	½
		OR	

Q. No.	Sub Q. N.	Answer	Marking Scheme
2	f)	$\int \frac{\sec^2 x}{(1+\tan x)(2+\tan x)}\,dx$	
		Put $\tan x = t$	
		$\sec^2 x\,dx = dt$	1
		$\therefore I = \int \frac{dt}{(1+t)(2+t)}$	
		$\therefore I = \int \frac{dt}{t^2+3t+2}$	½
		Third term $= \frac{(3)^2}{4} = \frac{9}{4}$	
		$\therefore I = \int \frac{dt}{t^2+3t+\frac{9}{4}-\frac{9}{4}+2}$	½
		$\therefore I = \int \frac{dt}{\left(t+\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2}$	½
		$\therefore I = \frac{1}{2\left(\frac{1}{2}\right)}\log\left(\frac{t+\frac{3}{2}-\frac{1}{2}}{t+\frac{3}{2}+\frac{1}{2}}\right)$	1
		$\therefore I = \log\left(\frac{t+1}{t+2}\right)$	
		$I = \log\left(\frac{1+\tan x}{2+\tan x}\right)+c$	½
3		**Attempt any <u>FOUR</u> of the following:**	**16**
	a)	Evaluate: $\int_0^4 \frac{1}{\sqrt{4x-x^2}}\,dx$	**04**
	Ans	$\int_0^4 \frac{1}{\sqrt{4x-x^2}}\,dx$	
		$= \int_0^4 \frac{1}{\sqrt{-(x^2-4x)}}\,dx$	
		Third term $= \frac{(4)^2}{4} = 4$	

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3	a)	$=\int_0^4 \frac{1}{\sqrt{-\left(x^2-4x+4-4\right)}}\,dx$	1
		$=\int_0^4 \frac{1}{\sqrt{-\left[\left(x-2\right)^2-\left(2\right)^2\right]}}\,dx$	
		$=\int_0^4 \frac{1}{\sqrt{\left(2\right)^2-\left(x-2\right)^2}}\,dx$	1
		$=\left[\sin^{-1}\left(\frac{x-2}{2}\right)\right]_0^4$	1
		$=\sin^{-1}\left(\frac{4-2}{2}\right)-\sin^{-1}\left(\frac{0-2}{2}\right)$	
		$=\sin^{-1}\left(1\right)-\sin^{-1}\left(-1\right)$	½
		$=\frac{\pi}{2}+\frac{\pi}{2}$	
		$=\pi$	½
		OR	
		$\int_0^4 \frac{1}{\sqrt{4x-x^2}}\,dx$	
		$\text{Third term}=\frac{\left(4\right)^2}{4}=4$	
		$=\int_0^4 \frac{1}{\sqrt{4-\left(4-4x+x^2\right)}}\,dx$	1
		$=\int_0^4 \frac{1}{\sqrt{\left(2\right)^2-\left(2-x\right)^2}}\,dx$	1
		$=\left[\sin^{-1}\left(\frac{2-x}{2}\right)\left(\frac{1}{-1}\right)\right]_0^4$	1
		$=-\sin^{-1}\left(\frac{2-4}{2}\right)+\sin^{-1}\left(\frac{2-0}{2}\right)$	
		$=-\sin^{-1}\left(-1\right)+\sin^{-1}\left(1\right)$	½
		$=\frac{\pi}{2}+\frac{\pi}{2}$	
		$=\pi$	½
		--	

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3	b)	Evaluate: $\int_0^{\pi} \frac{x.\sin x}{1+\cos^2 x}\, dx$	04
	Ans	$\int_0^{\pi} \frac{x.\sin x}{1+\cos^2 x}\, dx$	
		$I = \int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)}{1+\cos^2(\pi - x)}\, dx$	½
		$\therefore I = \int_0^{\pi} \frac{(\pi - x)\sin x}{1+\cos^2 x}\, dx$	½
		$\therefore I = \int_0^{\pi} \frac{\pi \sin x}{1+\cos^2 x}\, dx - \int_0^{\pi} \frac{x \sin x}{1+\cos^2 x}\, dx$	
		$\therefore I = \int_0^{\pi} \frac{\pi \sin x}{1+\cos^2 x}\, dx - I$	½
		$\therefore 2I = \int_0^{\pi} \frac{\pi \sin x}{1+\cos^2 x}\, dx$	
		Put $\cos x = t$ when $x \to 0$ to π, $t \to 1$ to -1	
		$\therefore -\sin x dx = dt$	½
		$\therefore \sin x dx = -dt$	½
		$\therefore 2I = -\pi \int_1^{-1} \frac{1}{1+t^2}\, dt$	½
		$\therefore 2I = -\pi \left[\tan^{-1} t\right]_1^{-1}$	½
		$\therefore 2I = -\pi \left[\tan^{-1}(-1) - \tan^{-1} 1\right]$	
		$\therefore 2I = -\pi \left[-\frac{\pi}{4} - \frac{\pi}{4}\right]$	
		$\therefore I = \frac{\pi^2}{4}$	½
	c)	Find the area of the loop of the curve $y^2 = x^2(1-x)$	04
	Ans	$y^2 = x^2(1-x)$	
		$\because y = 0$	
		$\therefore x^2(1-x) = 0$	
		$\therefore x = 0,1$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
3	c)	$\text{Area } A = \int_a^b y\,dx$	
		$\therefore A = \int_0^1 x\sqrt{1-x}\,dx$	½
		$\therefore A = \int_0^1 (1-x)\sqrt{x}\,dx$	
		$\therefore A = \int_0^1 (1-x)\,x^{\frac{1}{2}}\,dx$	½
		$\therefore A = \int_0^1 \left(x^{\frac{1}{2}} - x^{\frac{3}{2}}\right)dx$	½
		$\therefore A = \left[\dfrac{x^{\frac{3}{2}}}{\frac{3}{2}} - \dfrac{x^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^1$	1
		$\therefore A = \dfrac{2}{3} - \dfrac{2}{5}$	
		$\therefore A = \dfrac{4}{15}$ or 0.2667	½
		OR	
		$y^2 = x^2(1-x)$	
		$\because y = 0$	1
		$\therefore x^2(1-x) = 0 \qquad \therefore x = 0,1$	
		$y = x\sqrt{1-x}$	
		$\text{Area } A = \int_a^b y\,dx$	
		$\therefore A = \int_0^1 x\sqrt{1-x}\,dx$	½
		$\text{Put } 1-x = t \Rightarrow 1-t = x$ when $x \to 0$ to 1, $t \to 1$ to 0	
		$\therefore -dx = dt$	
		$\therefore dx = -dt$	½
		$\therefore A = \int_1^0 (1-t)\sqrt{t}\,(-dt)$	
		$\therefore A = -\int_1^0 (1-t)\,t^{\frac{1}{2}}\,dt$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
3	c)	$\therefore A = -\int_{1}^{0}\left(t^{\frac{1}{2}} - t^{\frac{3}{2}}\right)dt$	½
		$\therefore A = -\left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{t^{\frac{5}{2}}}{\frac{5}{2}}\right]_{1}^{0}$	1
		$\therefore A = 0 + \left(\frac{2}{3} - \frac{2}{5}\right)$	½
		$\therefore A = \frac{4}{15}$ or 0.2667	
		--	
	d)	Solve: $\frac{dy}{dx} = e^{(x-y)}.x^2$	**04**
	Ans	$\frac{dy}{dx} = e^{(x-y)}x^2$	
		$\therefore \frac{dy}{dx} = e^{x}e^{-y}x^2$	½
		$\therefore e^{y}dy = e^{x}x^2dx$	½
		$\therefore \int e^{y}dy = \int e^{x}x^2dx$	
		$\therefore e^{y} = x^2\int e^{x}dx - \int\left(\int e^{x}dx\,\frac{d}{dx}x^2\right)dx$	½
		$\therefore e^{y} = e^{x}x^2 - \int e^{x}2xdx$	1
		$\therefore e^{y} = e^{x}x^2 - 2\int e^{x}xdx$	
		$\therefore e^{y} = e^{x}x^2 - 2\left[e^{x}x - \int e^{x}dx\right]$	½
		$\therefore e^{y} = e^{x}x^2 - 2\left[xe^{x} - e^{x}\right] + c$	1
		--	
	e)	Solve: $(x-y)\frac{dy}{dx} = x + y$	**04**
	Ans	$(x-y)\frac{dy}{dx} = x + y$	
		$\therefore \frac{dy}{dx} = \frac{x+y}{x-y}$	
		Put $y = vx$	
		$\therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
3	e)	$\therefore v + x\dfrac{dv}{dx} = \dfrac{x+vx}{x-vx}$	½
		$\therefore v + x\dfrac{dv}{dx} = \dfrac{1+v}{1-v}$	
		$\therefore x\dfrac{dv}{dx} = \dfrac{1+v}{1-v} - v$	
		$\therefore x\dfrac{dv}{dx} = \dfrac{1+v-v+v^2}{1-v}$	
		$\therefore x\dfrac{dv}{dx} = \dfrac{1+v^2}{1-v}$	½
		$\therefore \dfrac{1-v}{1+v^2}dv = \dfrac{1}{x}dx$	½
		$\therefore \int \dfrac{1-v}{1+v^2}dv = \int \dfrac{1}{x}dx$	
		$\int \left(\dfrac{1}{1+v^2} - \dfrac{v}{1+v^2}\right) dv = \log x + c$	½
		$\tan^{-1} v - \dfrac{1}{2}\log\left(1+v^2\right) = \log x + c$	1
		$\tan^{-1}\left(\dfrac{y}{x}\right) - \dfrac{1}{2}\log\left(1+\dfrac{y^2}{x^2}\right) = \log x + c$	½
		---	
	f)	Solve: $(1+x)\dfrac{dy}{dx} - y = e^{3x}(1+x)^2$	**04**
	Ans	$(1+x)\dfrac{dy}{dx} - y = e^{3x}(1+x)^2$	
		$\therefore \dfrac{dy}{dx} - \dfrac{1}{1+x}y = e^{3x}(1+x)$	½
		$P = -\dfrac{1}{1+x}, Q = e^{3x}(1+x)$	½
		$I.F. = e^{\int -\frac{1}{1+x}dx} = e^{-\int \frac{1}{1+x}dx}$	
		$I.F. = e^{-\log(1+x)} = e^{\log\frac{1}{1+x}} = \dfrac{1}{1+x}$	1
		Solution is	
		$yI.F. = \int QI.F.dx + c$	
		$\therefore y\dfrac{1}{1+x} = \int e^{3x}(1+x)\dfrac{1}{1+x}dx + c$	½
		$\therefore \dfrac{y}{1+x} = \int e^{3x}dx + c$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
3	f)	$\therefore \frac{y}{1+x} = \frac{e^{3x}}{3} + c$	1
,		--	
4		**Attempt any FOUR of the following:**	**16**
	a)	Evaluate: $\int_1^4 \frac{\sqrt{5-x}}{\sqrt{x}+\sqrt{5-x}}\,dx$	**04**
	Ans	$I = \int_1^4 \frac{\sqrt{5-x}}{\sqrt{x}+\sqrt{5-x}}\,dx$ --------------------(1)	
		$I = \int_1^4 \frac{\sqrt{5-(1+4-x)}}{\sqrt{(1+4-x)}+\sqrt{5-(1+4-x)}}\,dx$	½
		$\therefore I = \int_1^4 \frac{\sqrt{5-5+x}}{\sqrt{5-x}+\sqrt{5-5+x}}\,dx$	
		$\therefore I = \int_1^4 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}\,dx$ --------------------(2)	½
		add (1) and (2)	
		$I + I = \int_1^4 \frac{\sqrt{5-x}}{\sqrt{x}+\sqrt{5-x}}\,dx + \int_1^4 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}\,dx$	
		$\therefore 2I = \int_1^4 \frac{\sqrt{5-x}+\sqrt{x}}{\sqrt{x}+\sqrt{5-x}}\,dx$	1
		$\therefore 2I = \int_1^4 1\,dx$	½
		$\therefore 2I = [x]_1^4$	½
		$\therefore 2I = 4-1$	
		$\therefore 2I = 3$	½
		$I = \frac{3}{2}$	½
		--	
	b)	Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\sin x.\cos x}{\cos^2 x + 3\cos x + 2}\,dx$	**04**
	Ans	$\int_0^{\frac{\pi}{2}} \frac{\sin x.\cos x}{\cos^2 x + 3\cos x + 2}\,dx$	

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Q. No.	Sub Q. N.	Answer	Marking Scheme
4	b)	Put $\cos x = t$ when $x \to 0$ to $\frac{\pi}{2}$, $t \to 1$ to 0	
		$\therefore -\sin x dx = dt$	1
		$\therefore \sin x dx = -dt$	
		$I = \int_1^0 \frac{t}{t^2+3t+2}(-dt)$	
		$\therefore I = -\int_1^0 \frac{t}{(t+1)(t+2)} dt$	½
		Let $\frac{t}{(t+1)(t+2)} = \frac{A}{t+1} + \frac{B}{t+2}$	
		$t = A(t+2) + B(t+1)$	
		Put $t = -1$	
		$-1 = A(1)$	½
		$\therefore A = -1$	
		Put $t = -2$	
		$-2 = B(-1)$	½
		$\therefore B = 2$	
		$\therefore \frac{t}{(t+1)(t+2)} = \frac{-1}{t+1} + \frac{2}{t+2}$	
		$\therefore -\int_1^0 \frac{t}{(t+1)(t+2)} dt = -\int_1^0 \left(\frac{-1}{t+1} + \frac{2}{t+2}\right) dt$	½
		$\therefore I = \left[\log(1+t) - 2\log(t+2)\right]_1^0$	½
		$\therefore I = (\log 1 - 2\log 2) - (\log 2 - 2\log 3)$	½
		$\therefore I = -3\log 2 + 2\log 3$	
		--	
	c)	Find the area bounded by two parabola $y^2 = 2x$ and $x^2 = 2y$	**04**
	Ans	$y^2 = 2x$ ---------(1)	
		$x^2 = 2y$	
		$\therefore y = \frac{x^2}{2}$	
		$eq.(1) \Rightarrow$	
		$\left(\frac{x^2}{2}\right)^2 = 2x$	

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Q. No.	Sub Q. N.	Answer	Marking Scheme
4	c)	$\frac{x^4}{4} = 2x$ $\therefore x^4 = 8x$ $\therefore x^4 - 8x = 0$ $\therefore x\left(x^3 - 8\right) = 0$ $\therefore x = 0, 2$ Area $A = \int_a^b (y_1 - y_2)\,dx$ $\therefore A = \int_0^2 \left(\sqrt{2x} - \frac{x^2}{2}\right)dx$ $\therefore A = \int_0^2 \left(\sqrt{2}\,x^{\frac{1}{2}} - \frac{x^2}{2}\right)dx$ $\therefore A = \left(\frac{\sqrt{2}\,x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^3}{6}\right)_0^2$ $\therefore A = \left(\frac{\sqrt{2}\,(2)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{(2)^3}{6}\right) - 0$ $\therefore A = \frac{4}{3}$ or 1.333 OR $y^2 = 2x$ ---------(1) $x^2 = 2y$ ---------(2) $\therefore x = \frac{y^2}{2}$ $eq.(2) \Rightarrow$ $\left(\frac{y^2}{2}\right)^2 = 2y$ $\frac{y^4}{4} = 2y$ $\therefore y^4 = 8y$ $\therefore y^4 - 8y = 0$ $\therefore y\left(y^3 - 8\right) = 0$ $\therefore y = 0, 2$	1 1 1 ½ ½ 1

Q. No.	Sub Q. N.	Answer	Marking Scheme
4	c)	Area $A=\int_a^b (x_1-x_2)\,dy$	
		$\therefore\ A=\int_0^2\left(\sqrt{2y}-\frac{y^2}{2}\right)dy$	1
		$\therefore\ A=\int_0^2\left(\sqrt{2}\,y^{\frac{1}{2}}-\frac{y^2}{2}\right)dy$	
		$\therefore\ A=\left(\frac{\sqrt{2}\,y^{\frac{3}{2}}}{\frac{3}{2}}-\frac{y^3}{6}\right)_0^2$	1
		$\therefore\ A=\left(\frac{\sqrt{2}\,(2)^{\frac{3}{2}}}{\frac{3}{2}}-\frac{(2)^3}{6}\right)-0$	½
		$\therefore\ A=\frac{4}{3}$ or 1.333	½
		--	

Q. No.	Sub Q. N.	Answer	Marking Scheme
4	d)	Solve: $\left[4-\frac{y^2}{x^2}\right]dx+\frac{2y}{x}dy=0$	**04**
	Ans	$\left[4-\frac{y^2}{x^2}\right]dx+\frac{2y}{x}dy=0$	
		$M=4-\frac{y^2}{x^2}\quad,\quad N=\frac{2y}{x}$	½
		$\frac{\partial M}{\partial y}=-\frac{2y}{x^2}\quad,\quad\frac{\partial N}{\partial x}=-\frac{2y}{x^2}$	1
		$\therefore\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$	½
		$\therefore$ exact D.E. is exact	
		Solution is	
		$\int_{y-constant}M\ dx+\int_{terms\ not\ containing\ 'x'}N\,dy=c$	1
		$\therefore\int_{y-constant}\left(4-\frac{y^2}{x^2}\right)dx+0=c$	
		$\therefore 4x+\frac{y^2}{x}=c$	1
		OR	
		$\left[4-\frac{y^2}{x^2}\right]dx+\frac{2y}{x}dy=0$	
		$\left[4-\frac{y^2}{x^2}\right]+\frac{2y}{x}\frac{dy}{dx}=0$	
		Put $\frac{y}{x}=v$	
		$\therefore y=vx$	
		$\therefore\frac{dy}{dx}=v+x\frac{dv}{dx}$	1
		$\therefore\left(4-v^2\right)+2v\left(v+x\frac{dv}{dx}\right)=0$	
		$\therefore 4-v^2+2v^2+2vx\frac{dv}{dx}=0$	
		$\therefore 4+v^2+2vx\frac{dv}{dx}=0$	
		$\therefore 2vx\frac{dv}{dx}=-\left(4+v^2\right)$	
		$\therefore\frac{2v}{4+v^2}dv=-\frac{1}{x}dx$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
4	d)	$\therefore \int \frac{2v}{4+v^2}dv = -\int \frac{1}{x}dx$	½
		$\log\left(4+v^2\right) = -\log x + c$	1
		$\log\left(4+\frac{y^2}{x^2}\right) = -\log x + c$	½
	e)	Solve: $\left(y.e^{xy}-2y^3\right)dx + \left(x.e^{xy}-6xy^2-2y\right)dy = 0$	**04**
	Ans	$\left(y.e^{xy}-2y^3\right)dx + \left(x.e^{xy}-6xy^2-2y\right)dy = 0$	
		$M = y.e^{xy}-2y^3 \quad , \quad N = x.e^{xy}-6xy^2-2y$	½
		$\frac{\partial M}{\partial y} = ye^{xy}x + e^{xy} - 6y^2 \quad , \quad \frac{\partial N}{\partial x} = xe^{xy}y + e^{xy} - 6y^2$	1
		$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$	½
		$\therefore$ D.E. exact	
		Solution is	
		$\int_{y-\text{constant}} M\ dx + \int_{\text{terms not containing 'x'}} N dy = c$	
		$\therefore \int_{y-\text{constant}} \left(y.e^{xy}-2y^3\right)\ dx + \int(-2y)\ dy = c$	½
		$\therefore y\frac{e^{xy}}{y} - 2y^3x - 2\frac{y^2}{2} = c$	1
		$\therefore e^{xy} - 2xy^3 - y^2 = c$	½
	f)	Verify that $y^2 = ax^2$ is a solution of $x\left(\frac{dy}{dx}\right)^2 - 2y\frac{dy}{dx} + ax = 0$	**04**
	Ans	$y^2 = ax^2$	
		$\therefore 2y\frac{dy}{dx} = 2ax$	
		$\therefore \frac{dy}{dx} = \frac{ax}{y}$	1
		consider	
		$L.H.S. = x\left(\frac{dy}{dx}\right)^2 - 2y\frac{dy}{dx} + ax$	
		$= x\left(\frac{ax}{y}\right)^2 - 2y\frac{ax}{y} + ax$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
4		$= x\frac{a^2x^2}{y^2} - 2ax + ax$	1
		$= x\frac{a^2x^2}{ax^2} - ax$	½
		$= ax - ax$	½
		$= 0 = R.H.S.$	½
		--	
5		**Attempt any FOUR of the following:**	**16**
	a)	The probabality that a student passes H.S.C. exam is 2/3 and the probability that he passes both H.S.C.and I.I.T. entrace exam is 14/45. The probability that he passes at least one exam is 4/5. What is the probability that he passes the I.I.T.entrance exam?	**04**
	Ans	Given $P(A) = \frac{2}{3}$	
		$P(A \cap B) = \frac{14}{45}$	
		$P(A \cup B) = \frac{4}{5}$	
		$P(B) = ?$	1
		$\because\ P(A \cup B) = P(A) + P(B) - P(A \cap B)$	
		$\therefore\ \frac{4}{5} = \frac{2}{3} + P(B) - \frac{14}{45}$	1
		$P(B) = \frac{4}{5} - \frac{2}{3} + \frac{14}{45}$	1
		$\therefore\ P(B) = \frac{4}{9}$ or 0.444	1
		--	
	b)	In 200 sets of tosses of 5 fair coins in how many ways you can expect $i)$ at least two heads. $ii)$ At the most two heads.	**04**
	Ans	$p = \frac{1}{2}$, $q = \frac{1}{2}$	
		$n = 5$	
		$p(r) = {}^nC_r p^r q^{n-r}$	
		$i)$ at least two heads	
		$P(r) = 1 - \left[p(0) + p(1) \right]$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
5	b)	$= 1 - \left[{}^5C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{5-0} + {}^5C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{5-1} \right]$	1
		$= \frac{13}{16}$ or 0.8125	½
		$\text{No. of ways} = 200 \times 0.8125 = 162.5 \approx 163$	½
		$ii)$ At the most two heads	
		$P(r) = p(0) + p(1) + p(2)$	1
		$= 0.1875 + {}^5C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{5-2}$	
		$= 0.5$	½
		$\text{No. of ways} = 200 \times 0.5 = 100$	½
		--	
	c)	If 5% of the electric bulbs manufacturing by a company are defective, use Poisson distribution to find the probability that in a sample of 100 bulbs.	**04**
		$i)$ None is defective	
		$ii)$ Five bulbs are defective (Given $e^{-5} = 0.007$)	
	Ans	$p = 5\% = 0.05$	
		$n = 100$	
		mean $m = np = 100 \times 0.05$	
		$m = 5$	1
		$P(r) = \frac{e^{-m} m^r}{r!}$	
		$i)$ None is defective	
		$r = 0$	1
		$P(0) = \frac{e^{-5}(5)^0}{0!}$	½
		$P(0) = 0.007$	
		$ii)$ Five bulbs are defective	
		$r = 5$	
		$P(5) = \frac{e^{-5}(5)^5}{5!}$	1
		$P(5) = 0.1823$	½
		--	
	d)	Evaluate: $\int \frac{x+1}{(x-1)^2} dx$	**04**

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Q. No.	Sub Q. N.	Answer	Marking Scheme
5	d) Ans	$\int \frac{x+1}{(x-1)^2}\,dx$	
		$= \int \frac{x-1+2}{(x-1)^2}\,dx$	1
		$= \int \left(\frac{x-1}{(x-1)^2} + \frac{2}{(x-1)^2}\right) dx$	1
		$= \int \left(\frac{1}{x-1} + \frac{2}{(x-1)^2}\right) dx$	½
		$= \log(x-1) + \frac{2(x-1)^{-1}}{-1} + c$	1
		$= \log(x-1) - \frac{2}{x-1} + c$	½
		OR	
		$\int \frac{x+1}{(x-1)^2}\,dx$	
		Let $\frac{x+1}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2}$	½
		$x+1 = A(x-1)+B$	
		Put $x=1$	
		$2 = B$	
		$\therefore B = 2$	½
		Put $x = 0$	
		$1 = A(-1)+B$	
		$1 = A(-1)+2$	½
		$\therefore A = 1$	
		$\therefore \frac{x+1}{(x-1)^2} = \frac{1}{x-1} + \frac{2}{(x-1)^2}$	
		$\int \frac{x+1}{(x-1)^2}\,dx = \int \left(\frac{1}{x-1} + \frac{2}{(x-1)^2}\right) dx$	1
		$= \int \left(\frac{1}{x-1} + \frac{2}{(x-1)^2}\right) dx$	
		$= \log(x-1) + \frac{2(x-1)^{-1}}{-1} + c$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
5	d)	$= \log(x-1) - \frac{2}{x-1} + c$	½
		OR	
		$\int \frac{x+1}{(x-1)^2}\,dx$	
		$= \int \frac{x+1}{x^2-2x+1}\,dx$	
		$= \frac{1}{2}\int \frac{2x+2}{x^2-2x+1}\,dx$	1
		$= \frac{1}{2}\int \frac{2x-2+4}{x^2-2x+1}\,dx$	
		$= \frac{1}{2}\left(\int \frac{2x-2}{x^2-2x+1}\,dx + 4\int \frac{1}{x^2-2x+1}\,dx\right)$	½
		$= \frac{1}{2}\left(\int \frac{2x-2}{x^2-2x+1}\,dx + 4\int \frac{1}{(x-1)^2}\,dx\right)$	½
		$= \frac{1}{2}\left(\log(x^2-2x+1) + 4\frac{(x-1)^{-1}}{-1}\right) + c$	1
		$= \frac{1}{2}\left(\log(x^2-2x+1) - \frac{4}{x-1}\right) + c$	1
		--	
	e)	Evaluate: $\int_0^{\frac{\pi}{2}} \sin 5x \cos 3x\,dx$	**04**
	Ans	$\int_0^{\frac{\pi}{2}} \sin 5x \cos 3x\,dx$	
		$= \frac{1}{2}\int_0^{\frac{\pi}{2}} 2\sin 5x \cos 3x\,dx$	½
		$= \frac{1}{2}\int_0^{\frac{\pi}{2}} \left(\sin(5x+3x) + \sin(5x-3x)\right)\,dx$	1
		$= \frac{1}{2}\int_0^{\frac{\pi}{2}} (\sin 8x + \sin 2x)\,dx$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
5	e)	$=\frac{1}{2}\left[\frac{-\cos 8x}{8}-\frac{\cos 2x}{2}\right]_0^{\frac{\pi}{2}}$	1
		$=-\frac{1}{2}\left[\frac{\cos 8x}{8}+\frac{\cos 2x}{2}\right]_0^{\frac{\pi}{2}}$	
		$=-\frac{1}{2}\left[\frac{\cos 8\left(\frac{\pi}{2}\right)}{8}+\frac{\cos 2\left(\frac{\pi}{2}\right)}{2}-\frac{\cos 0}{8}-\frac{\cos 0}{2}\right]$	1
		$=-\frac{1}{2}\left[\frac{\cos 4\pi}{8}+\frac{\cos \pi}{2}-\frac{\cos 0}{8}-\frac{\cos 0}{2}\right]$	
		$=-\frac{1}{2}\left[\frac{1}{8}+\frac{(-1)}{2}-\frac{1}{8}-\frac{1}{2}\right]$	
		$=\frac{1}{2}$	½
		--	
	f)	Evaluate: $\int e^x.\sin 4x\ dx$	**04**
	Ans	$\int e^x \sin 4x\ dx$	
		$I=\int e^x \sin 4x\ dx$	
		$\therefore\ I=\sin 4x\int e^x dx-\int\left(\int e^x dx.\frac{d}{dx}\sin 4x\right)dx$	½
		$\therefore\ I=\sin 4x e^x-\int e^x \cos 4x.4\ dx$	1
		$\therefore\ I=\sin 4x e^x-4\int e^x \cos 4x\ dx$	
		$\therefore\ I=\sin 4x e^x-4\left[\cos 4x e^x-\int\left(\int e^x dx.\frac{d}{dx}\cos 4x\right)dx\right]$	½
		$\therefore\ I=\sin 4x e^x-4\left[\cos 4x e^x-\int e^x(-\sin 4x.4)\ dx\right]$	1
		$\therefore\ I=\sin 4x e^x-4\left[\cos 4x e^x+4\int e^x \sin 4x\ dx\right]$	
		$\therefore\ I=\sin 4x e^x-4e^x\cos 4x-16\int e^x \sin 4x\ dx$	
		$\therefore\ I=\sin 4x e^x-4e^x\cos 4x-16I$	½
		$\therefore 17I=\sin 4x e^x-4e^x\cos 4x$	
		$\therefore\ I=\frac{e^x}{17}(\sin 4x-4\cos 4x)+c$	½
		--	

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Q. No.	Sub Q. N.	Answer	Marking Scheme
6		**Attempt any FOUR of the following:**	**16**
	a)	Two six face unbaised dice are thrown .Find the probability that the sum of the numbers shown is 7 or product is 12.	**04**
	Ans	$S=\{(1,1)(1,2)(1,3)(1,4)(1,5)(1,6)$ $(2,1)(2,2)(2,3)(2,4)(2,5)(2,6)$ $(3,1)(3,2)(3,3)(3,4)(3,5)(3,6)$ $(4,1)(4,2)(4,3)(4,4)(4,5)(4,6)$ $(5,1)(5,2)(5,3)(5,4)(5,5)(5,6)$ $(6,1)(6,2)(6,3)(6,4)(6,5)(6,6)\}$	1
		$n(s)=36$ sum is 7 or product is 12 $\therefore A=\{(1,6)(2,5)(3,4)(4,3)(2,6)(5,2)(6,1)(6,2)\}$ $\therefore n(A)=8$	1
		$p(A)=\frac{n(A)}{n(s)}$ $p(A)=\frac{8}{36}$ or 0.222	2
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	b)	In a sample of 1000 cases , the mean of certain test is 14 and standard deviation is 2.5. Assuming the distribution is normal. Find $i)$ How many students score between 12 and 15? $ii)$ How many students score above 18? (Given: $A(0.8)=0.2881$, $A(0.4)=0.1554$, $A(1.6)=0.4452$)	**04**
	Ans	Given $\bar{x}=14, \sigma=2.5$ $z=\frac{x-\bar{x}}{\sigma}$ $i)$ For $x=12$ $z=\frac{12-14}{2.5}=-0.8$ For $x=12$ $z=\frac{15-14}{2.5}=0.4$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
6	b)	$p = (\text{area between } -0.8 \text{ and } 0.4) = A(-0.8) + A(0.4)$ $= 0.2881 + 0.1554$ $= 0.4435$ $\therefore$ No. of students $= 1000 \times 0.4435 = 443.5 \approx 444$ ii) For $x = 18$, $z = \dfrac{18-14}{2.5} = 1.6$ $p = (\text{area above } 1.6) = 0.5 - A(1.6)$ $= 0.5 - 0.4452$ $= 0.0548$ $\therefore$ No. of students $= 1000 \times 0.0548 = 54.8 \approx 55$	1 ½ ½ 1 ½
	c)	A metal wire 40 cm long is bent to form a rectangle. Find its dimensions when its area is maximum.	**04**
	Ans	Let length $= y$, breadth $= x$ Perimeter is $2x + 2y = 40$ $\therefore x + y = 20$ $y = 20 - x$ Area is $A = xy$ $\therefore A = x(20 - x)$ $\therefore A = 20x - x^2$ $\therefore \dfrac{dA}{dx} = 20 - 2x$ $\therefore \dfrac{d^2A}{dx^2} = -2$ Consider $\dfrac{dA}{dx} = 0$ $\therefore 20 - 2x = 0$ $\therefore x = 10$ $\therefore at\ x = 10$ $\dfrac{d^2A}{dx^2} = -2 < 0$ $\therefore A$ is maximum when $x = 10$ $\therefore$ breadth $x = 10$ length $y = 10$	½ 1 ½ ½ ½ ½ ½

WINTER – 16 EXAMINATION

Model Answer Subject Code: 17301

Q. No.	Sub Q. N.	Answer	Marking Scheme
6	d)	Find the equation of normal and tangent to the curve $y = 4.x.e^x$ at origin.	**04**
	Ans	$y = 4xe^x$	
		$\therefore \frac{dy}{dx} = 4\left(xe^x + e^x\right)$	½
		at $(0,0)$	
		$\therefore \frac{dy}{dx} = 4\left(e^0\right) = 4$	½
		$\therefore$ slope of tangent $m = 4$	½
		Equation of tangent is	
		$y - y_1 = m\left(x - x_1\right)$	
		$y - 0 = 4\left(x - 0\right)$	
		$y = 4x$	1
		$4x - y = 0$	
		$\therefore$ slope of normal $= -\frac{1}{\frac{dy}{dx}} = -\frac{1}{m} = -\frac{1}{4}$	½
		$y - 0 = -\frac{1}{4}\left(x - 0\right)$	
		$4y = -x$	
		$x + 4y = 0$	1
		--	
	e)	If $P(A) = \frac{1}{2}, P(B') = \frac{2}{3}, P(A \cup B) = \frac{2}{3}$, find $P(A' \cap B')$ & $P(A/B)$	**04**
	Ans	$P(A' \cap B') = P(A \cup B)'$	
		$= 1 - P(A \cup B)$	½
		$= 1 - \frac{2}{3}$	
		$= \frac{1}{3}$ or 0.333	½
		$P(B) = 1 - P(B')$	
		$= 1 - \frac{2}{3}$	
		$= \frac{1}{3}$	½
		$P(A \cup B) = P(A) + P(B) - P(A \cap B)$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
6	e)	$\frac{2}{3}=\frac{1}{2}+\frac{1}{3}-P(A\cap B)$	½
		$\therefore P(A\cap B)=\frac{1}{2}+\frac{1}{3}-\frac{2}{3}$	
		$\therefore P(A\cap B)=\frac{1}{2}-\frac{1}{3}$	
		$\therefore P(A\cap B)=\frac{1}{6}$	½
		$P(A/B)=\frac{P(A\cap B)}{P(B)}$	
		$=\frac{1/6}{1/3}$	1
		$=\frac{1}{2}$ or 0.5	½
		--	
	f)	The probability that a pen manufactured by a company will be defective is 1/10. If 12 such pens are manufactured, find the probability that:	**04**
		$i)$ Exactly two will be defective	
		$ii)$ At least two will be defective	
		$iii)$ None will be defective	
	Ans	Given $p=1/10=0.1$	
		$q=1-p=0.9$	½
		$n=12$	
		$p(r)={}^{n}C_{r}p^{r}q^{n-r}$	
		$i)$ Exactly two will be defective, $r=2$	
		$p(2)={}^{12}C_{2}(0.1)^{2}(0.9)^{12-2}$	
		$p(2)=0.2301$	1
		$ii)$ At least two will be defective	
		$1-[p(0)+p(1)]$	
		$=1-\left[{}^{12}C_{0}(0.1)^{0}(0.9)^{12-0}+{}^{12}C_{1}(0.1)^{1}(0.9)^{12-1}\right]$	1
		$=0.3409$	½
		$iii)$ None will be defective, $r=0$	
		$p(0)={}^{12}C_{0}(0.1)^{0}(0.9)^{12-0}$	
		$p(0)=0.2824$	1
		--	

Q. No.	Sub Q. N.	Answer	Marking Scheme
		Important Note *In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.* --- --	