



Winter – 2013 Examination

Subject & Code: Applied Maths (17301)

Model Answer

Page No: 1/26

| Que. No. | Sub. Que. | Model answers | Marks | Total Marks |
|----------|-----------|---|-------|-------------|
| | | <p>Important Instructions to the Examiners:</p> <p>1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.</p> <p>2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.</p> <p>3) The language errors such as grammatical, spelling errors should not be given more importance. (Not applicable for subject English and Communication Skills.)</p> <p>4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by the candidate and those in the model answer may vary. The examiner may give credit for any equivalent figure drawn.</p> <p>5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and the model answer.</p> <p>6) In case of some questions credit may be given by judgment on part of examiner of relevant answer based on candidate's understanding.</p> <p>7) For programming language papers, credit may be given to any other program based on equivalent concept.</p> | | |



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| 1) | | Attempt any TEN of the following: | | |
| | a) Ans. | <p>Find the point on the curve $y = 3x - x^2$ at which slope is -5.</p> $y = 3x - x^2$ $\therefore \frac{dy}{dx} = 3 - 2x$ <p>But given that, slope $m = -5$</p> $\therefore 3 - 2x = -5$ $\therefore -2x = -5 - 3 = -8$ $\therefore x = 4$ $\therefore y = 3(4) - (4)^2 = -4$ $\therefore \text{the point is } (4, -4)$ | $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ | 2 |
| | b) Ans. | <p>Find the radius of curvature of the curve $y = \log(\sin x)$ at $x = \frac{\pi}{2}$.</p> $y = \log(\sin x)$ $\therefore \frac{dy}{dx} = \frac{1}{\sin x} \cdot \cos x = \cot x$ $\& \frac{d^2y}{dx^2} = -\cos x c^2 x$ <p>$\therefore \text{at } x = \frac{\pi}{2}$,</p> $\frac{dy}{dx} = \cot\left(\frac{\pi}{2}\right) = 0$ <p>and $\frac{d^2y}{dx^2} = -\cos x c^2\left(\frac{\pi}{2}\right) = -1$</p> $\therefore \rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = \frac{\left[1 + (0)^2\right]^{\frac{3}{2}}}{-1} = -1$ | $\frac{1}{2}$ $\frac{1}{2}$ 1 | 2 |
| | c) Ans. | <p>Integrate w. r. t. x of $\int \sqrt{1 + \cos 2x} dx$</p> $\int \sqrt{1 + \cos 2x} dx = \int \sqrt{2 \cos^2 x} dx$ $= \sqrt{2} \int \cos x dx$ $= \sqrt{2} \sin x + c$ | $\frac{1}{2}$ $\frac{1}{2}$ 1 | 2 |
| | | Note: In the solution of any integration problems , if the constant c is not added, $\frac{1}{2}$ mark may be deducted. | | |



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| 1) | d) | Evaluate $\int \frac{\cos(\log x)}{x} dx$ | | |
| | Ans. | $\int \frac{\cos(\log x)}{x} dx$ <div style="display: flex; align-items: center; justify-content: space-between;"> $\int \cos t dt$ <div style="margin-right: 20px;"> $\left \begin{array}{l} \text{Put } \log x = t \\ \therefore \frac{1}{x} dx = dt \end{array} \right.$ </div> </div> $= \int \cos t dt$ $= \sin t + c$ $= \sin(\log x) + c$ | $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ | 2 |
| | | OR | | |
| | | $I = \int \frac{\cos(\log x)}{x} dx$ $\text{Put } \log x = t$ $\therefore \frac{1}{x} dx = dt$ $\therefore I = \int \cos t dt$ $= \sin t + c$ $= \sin(\log x) + c$ | $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ | 2 |
| | e) | Evaluate $\int \frac{dx}{x(x+1)}$ | | |
| | Ans. | $\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$ $\therefore [1 = (x+1)A + xB]$ $\text{Put } x=0$ $\therefore 1 = (0+1)A + 0$ $\therefore [1 = A]$ $\text{Put } x+1=0 \quad \therefore x=-1,$ $\therefore 1 = 0 + (-1)B$ $\therefore [-1 = B]$ $\therefore \frac{1}{x(x+1)} = \frac{1}{x} + \frac{-1}{x+1}$ $\int \frac{dx}{x(x+1)} = \int \left[\frac{1}{x} + \frac{-1}{x+1} \right] dx$ $= \log x - \log(x+1) + c$ | $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ | 2 |
| | | OR | | |



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Subject & Code: Applied Maths (17301)

Page No: 4/26

| Que. No. | Sub. Que. | Model answers | Marks | Total Marks |
|----------|-----------|---|---------------------------------|-------------|
| 1) | | $\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$ $\therefore [1 = (x+1)A + xB]$ <p>For $x=0$</p> $A = \frac{1}{x+1} = \frac{1}{0+1} = 1$ <p>For $x+1=0 \quad \therefore x=-1,$</p> $B = \frac{1}{x} = \frac{1}{-1} = -1$ $\therefore \frac{1}{x(x+1)} = \frac{1}{x} + \frac{-1}{x+1}$ $\int \frac{dx}{x(x+1)} = \int \left[\frac{1}{x} + \frac{-1}{x+1} \right] dx$ $= \log x - \log(x+1) + c$ | 1/2 1/2 1/2 + 1/2 | 2 |
| f) | | Evaluate $\int \tan^{-1} x dx$ | | |
| Ans. | | $\int \tan^{-1} x dx = \int 1 \cdot \tan^{-1} x dx$ $= \tan^{-1} x \int 1 dx - \int \left(\int 1 dx \right) \frac{d}{dx} (\tan^{-1} x) dx$ $= x \tan^{-1} x - \int x \cdot \frac{1}{x^2+1} \cdot dx$ $= x \tan^{-1} x - \frac{1}{2} \int \frac{2x}{x^2+1} \cdot dx$ $= x \tan^{-1} x - \frac{1}{2} \log(x^2+1) + c$ | 1/2 1/2 1/2 1/2 1/2 | 2 |
| g) | | Evaluate $\int_1^2 \frac{1}{3x-2} dx$ | | |
| Ans. | | $\int_1^2 \frac{1}{3x-2} dx = \left[\frac{\log(3x-2)}{3} \right]_1^2$ $= \frac{\log[3(2)-2]}{3} - \frac{\log[3(1)-2]}{3}$ $= \frac{\log 4}{3} - \frac{\log 1}{3}$ $= \frac{1}{3} \log 4$ | 1 1/2 1/2 | 2 |



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|----------|-----------|---|--------------------------|-------------|
| 1) | h) | <p>Find the area contained by the curve $y = 1 + x^3 + 2 \sin x$ from $x = 0$ to $x = \pi$.</p> $A = \int_a^b y dx$ $= \int_0^\pi (1 + x^3 + 2 \sin x) dx$ $= \left[x + \frac{x^4}{4} - 2 \cos x \right]_0^\pi$ $= \left[\pi + \frac{\pi^4}{4} - 2 \cos \pi \right] - [0 + 0 - 2 \cos 0]$ $= \pi + \frac{\pi^4}{4} + 4 \quad \text{or} \quad 31.494$ | 1/2 1/2 1/2 1/2 | 2 |
| | i) | <p>Find the order and degree of the D. E. $\frac{d^2 y}{dx^2} = \sqrt{y - \frac{dy}{dx}}$.</p> <p>Order = 2.</p> | 1 | |
| | Ans. | $\frac{d^2 y}{dx^2} = \sqrt{y - \frac{dy}{dx}}$ $\therefore \left(\frac{d^2 y}{dx^2} \right)^2 = y - \frac{dy}{dx}$ <p>Degree = 2</p> | 1/2 1/2 | 2 |
| | j) | <p>Form a differential equation if $y = A \sin x + B \cos x$.</p> <p>$y = A \sin x + B \cos x$</p> | | |
| | Ans. | $\therefore \frac{dy}{dx} = A \cos x - B \sin x$ $\therefore \frac{d^2 y}{dx^2} = -A \sin x - B \cos x$ $= -(A \sin x + B \cos x)$ $= -y$ $\therefore \frac{d^2 y}{dx^2} + y = 0$ | 1/2 1/2 1/2 1/2 | 2 |
| | k) | <p>From a pack of 52 cards one is drawn at random. Find the probability of getting a king.</p> <p>$p = \frac{m}{n} = \frac{4}{52}$</p> $= \frac{1}{13} \quad \text{or} \quad 0.0769$ | 1½ | |
| | Ans. | OR | 1/2 | 2 |



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|----------|------------|--|--|-------------|
| 1) | | $n = n(S) = 52$ $m = n(A) = 4$ $\therefore p = \frac{m}{n} = \frac{4}{52}$ $= \frac{1}{13} \text{ or } 0.0769$ <p style="text-align: center;">OR</p> $\therefore p = \frac{m}{n} = \frac{^4C_1}{^{52}C_1}$ $= \frac{1}{13} \text{ or } 0.0769$ | $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ | 2 |
| | l) Ans. | An unbiased coin is tossed 5 times. Find the probability of getting 3 heads. $p = \frac{1}{2} = 0.5$ $\therefore q = 1 - p = \frac{1}{2} = 0.5$ <i>Here n = 5</i> $p = {}^nC_r p^r q^{n-r}$ $= {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2$ $= \frac{5}{16} \text{ or } 0.3125$ | $\frac{1}{2}$ | |
| 2) | a) | Attempt any FOUR of the following: Find the equation of the tangent and normal to the curve $13x^3 + 2x^2y + y^3 = 1$ at $(1, -2)$. Ans. $13x^3 + 2x^2y + y^3 = 1$ $\therefore 39x^2 + 2\left(x^2 \frac{dy}{dx} + y \cdot 2x\right) + 3y^2 \frac{dy}{dx} = 0$ $\therefore 39x^2 + 2x^2 \frac{dy}{dx} + 4xy + 3y^2 \frac{dy}{dx} = 0$ $\therefore (2x^2 + 3y^2) \frac{dy}{dx} = -39x^2 - 4xy$ $\therefore \frac{dy}{dx} = \frac{-39x^2 - 4xy}{2x^2 + 3y^2}$ \therefore the slope of tangent at $(1, -2)$ is $m = \frac{dy}{dx} = \frac{-39(1)^2 - 4(1)(-2)}{2(1)^2 + 3(-2)^2} = -\frac{31}{14}$ | $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ | 2 |



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| 2) | | <p>∴ the equation of tangent is</p> $y + 2 = -\frac{31}{14}(x - 1)$ <p>∴ $14y + 28 = -31x + 31$</p> <p>∴ $[31x + 14y - 3 = 0] \quad \text{or} \quad [31x + 14y = 3]$</p> <p>∴ the slope of normal $= -\frac{1}{m} = \frac{14}{31}$</p> <p>∴ the equation of tangent is</p> $y + 2 = \frac{14}{31}(x - 1)$ <p>∴ $31y + 62 = 14x - 14$</p> <p>∴ $[14x - 31y - 76 = 0] \quad \text{or} \quad [-14x + 31y + 76 = 0]$</p> | $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ | |
| b) | | A beam is bent in the form of the curve $y = 2 \sin x - \sin 2x$. Find the radius of curvature of the beam at this point at $x = \frac{\pi}{2}$. | | 4 |
| Ans. | | $y = 2 \sin x - \sin 2x$ $\therefore \frac{dy}{dx} = 2 \cos x - 2 \cos 2x$ $\text{&} \frac{d^2y}{dx^2} = -2 \sin x + 4 \sin 2x$ $\therefore \text{at } x = \frac{\pi}{2},$ $\frac{dy}{dx} = 2 \cos\left(\frac{\pi}{2}\right) - 2 \cos 2\left(\frac{\pi}{2}\right) = 2$ $\text{and } \frac{d^2y}{dx^2} = -2 \sin\left(\frac{\pi}{2}\right) + 4 \sin 2\left(\frac{\pi}{2}\right) = -2$ $\therefore \rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = \frac{\left[1 + (2)^2\right]^{\frac{3}{2}}}{-2} = -\frac{5^{3/2}}{2} \quad \text{or} \quad -\frac{5\sqrt{5}}{2} \quad \text{or} \quad -5.590$ | $\frac{1}{2}$ $\frac{1}{2}$ 1 1 1 | 4 |
| c) | | Find the maximum and minimum values of $x^3 - 18x^2 + 96x$. Let $y = x^3 - 18x^2 + 96x$ $\therefore \frac{dy}{dx} = 3x^2 - 36x + 96$ $\therefore \frac{d^2y}{dx^2} = 6x - 36$ | $\frac{1}{2}$ $\frac{1}{2}$ | |



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| 2) | | <p>For stationary values, $\frac{dy}{dx} = 0$</p> $\therefore 3x^2 - 36x + 96 = 0$ $\therefore x = 4, 8$ <p>At $x = 4$, $\frac{d^2y}{dx^2} = 6(4) - 36 = -12 < 0$</p> <p>$\therefore$ At $x = 4$, y has maximum value and it is</p> $y = (4)^3 - 18(4)^2 + 96(4) = 160$ <p>At $x = 8$, $\frac{d^2y}{dx^2} = 6(8) - 36 = 12 > 0$</p> <p>$\therefore$ At $x = 8$, y has minimum value and it is</p> $y = (8)^3 - 18(8)^2 + 96(8) = 128$ | 1 1/2 1/2 1/2 | 4 |
| d) | | Evaluate $\int \frac{1-\tan x}{1+\tan x} dx$ | | |
| Ans. | | $\int \frac{1-\tan x}{1+\tan x} dx = \int \tan\left(\frac{\pi}{4} - x\right) dx$ $= -\log \sec\left(\frac{\pi}{4} - x\right) + c$ | 2 2 | 4 |
| | | OR | | |
| | | $\int \frac{1-\tan x}{1+\tan x} dx = \int \frac{1 - \frac{\sin x}{\cos x}}{1 + \frac{\sin x}{\cos x}} dx$ $= \int \frac{\cos x - \sin x}{\cos x + \sin x} dx$ $= \log(\cos x + \sin x) + c$ | 2 2 | 4 |
| | | OR | | |
| | | $\int \frac{1-\tan x}{1+\tan x} dx = \int \frac{1 - \frac{\sin x}{\cos x}}{1 + \frac{\sin x}{\cos x}} dx$ $= \int \frac{\cos x - \sin x}{\cos x + \sin x} dx$ | | |
| | | $\boxed{\begin{aligned} &\text{Put } \cos x + \sin x = t \\ &\therefore (-\sin x + \cos x) dx = dt \end{aligned}}$ | 1+1 | |
| | | $= \int \frac{1}{t} dx$ $= \log t + c$ $= \log(\cos x + \sin x) + c$ | 1/2 1 1/2 | 4 |



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| 2) | e) | Evaluate $\int \sin^3 x dx$ | | |
| | Ans. | $\int \sin^3 x dx = \int \frac{1}{4} (3 \sin x - \sin 3x) dx$ $= \frac{1}{4} \left(-3 \cos x + \frac{\cos 3x}{3} \right) + c$ <p style="text-align: center;">OR</p> $\int \sin^3 x dx = \int \sin^2 x \cdot \sin x dx$ $= \int (1 - \cos^2 x) \sin x dx$ <div style="border: 1px solid black; padding: 5px; display: inline-block;"> $\boxed{\begin{array}{l} \text{Put } \cos x = t \\ \therefore -\sin x dx = dt \end{array}}$ </div> $= - \int (1 - t^2) dt$ $= - \left(t - \frac{t^3}{3} \right) + c$ $= - \left(\cos x - \frac{\cos^3 x}{3} \right) + c$ | 2 2 1 1 1 1 | 4 |
| | f) | Evaluate $\int \frac{x+1}{x(x^2-4)} dx$ | | |
| | Ans. | $I = \int \frac{x+1}{x(x^2-4)} dx = \int \frac{x+1}{x(x-2)(x+2)} dx$ $\frac{x+1}{x(x-2)(x+2)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2}$ <p>For $x=0$</p> $A = \frac{x+1}{(x-2)(x+2)} = \frac{0+1}{(0-2)(0+2)} = \boxed{-\frac{1}{4}}$ <p>For $x-2=0$ $\therefore x=2$,</p> $B = \frac{x+1}{x(x+2)} = \frac{2+1}{2(2+2)} = \boxed{\frac{3}{8}}$ <p>For $x+2=0$ $\therefore x=-2$,</p> $C = \frac{x+1}{x(x-2)} = \frac{-2+1}{-2(-2+2)} = \boxed{-\frac{1}{8}}$ $\therefore \frac{x+1}{x(x-2)(x+2)} = \frac{-1/4}{x} + \frac{3/8}{x-2} + \frac{-1/8}{x+2}$ $I = \int \left[\frac{-1/4}{x} + \frac{3/8}{x-2} + \frac{-1/8}{x+2} \right] dx$ $= -\frac{1}{4} \log x + \frac{3}{8} \log(x-2) - \frac{1}{8} \log(x+2) + c$ <p style="text-align: center;">OR</p> | 1 1 1 1 1 1 | 4 |



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| 2) | | <p style="text-align: center;">OR</p> $I = \int \frac{x+1}{x(x^2-4)} dx = \int \frac{x+1}{x(x-2)(x+2)} dx$ $\frac{x+1}{x(x-2)(x+2)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2}$ $\therefore \boxed{x+1 = (x-2)(x+2)A + x(x+2)B + x(x-2)C}$ <p>Put $x = 0$</p> $\therefore 0+1 = (0-2)(0+2)A + 0 + 0$ $\therefore \boxed{-\frac{1}{4} = A}$ <p>Put $x-2 = 0$ $\therefore x = 2,$</p> $\therefore 2+1 = 0+2(2+2)B + 0$ $\therefore \boxed{\frac{3}{8} = B}$ <p>Put $x+2 = 0$ $\therefore x = -2,$</p> $\therefore -2+1 = 0+0-2(-2-2)C$ $\therefore \boxed{-\frac{1}{8} = C}$ $\therefore \frac{x+1}{x(x-2)(x+2)} = \frac{-1/4}{x} + \frac{3/8}{x-2} + \frac{-1/8}{x+2}$ $I = \int \left[\frac{-1/4}{x} + \frac{3/8}{x-2} + \frac{-1/8}{x+2} \right] dx$ $= -\frac{1}{4} \log x + \frac{3}{8} \log(x-2) - \frac{1}{8} \log(x+2) + c$ | 1 1 1 | 4 |
| 3) | a) | <p>Attempt any FOUR of the following.</p> <p>Evaluate $\int_0^4 \frac{dx}{\sqrt{4x-x^2}}$</p> <p>Ans.</p> $4x-x^2 = 4-4+4x-x^2 = 4-(x-2)^2$ $= 2^2 - (x-2)^2$ $\int_0^4 \frac{dx}{\sqrt{4x-x^2}} = \int_0^4 \frac{dx}{\sqrt{2^2-(x-2)^2}}$ $= \left[\sin^{-1} \left(\frac{x-2}{2} \right) \right]_0^4$ $= \sin^{-1} \left(\frac{4-2}{2} \right) - \sin^{-1} \left(\frac{0-2}{2} \right)$ $= \sin^{-1}(1) - \sin^{-1}(-1)$ $= \frac{\pi}{2} + \frac{\pi}{2} = \pi$ | 1 1 1 1 | 4 |



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| 3) | | <p style="text-align: center;">OR</p> $\begin{aligned} \int_0^4 \frac{dx}{\sqrt{4x-x^2}} &= \int_0^4 \frac{dx}{\sqrt{4-4+4x-x^2}} \\ &= \int_0^4 \frac{dx}{\sqrt{4-(x-2)^2}} \\ &= \int_0^4 \frac{dx}{\sqrt{2^2-(x-2)^2}} \\ &= \left[\sin^{-1}\left(\frac{x-2}{2}\right) \right]_0^4 \\ &= \sin^{-1}\left(\frac{4-2}{2}\right) - \sin^{-1}\left(\frac{0-2}{2}\right) \\ &= \sin^{-1}(1) - \sin^{-1}(-1) \\ &= \frac{\pi}{2} + \frac{\pi}{2} = \pi \end{aligned}$ <hr/> <p>b) Evaluate $\int_0^5 \frac{\sqrt{9-x}}{\sqrt{9-x} + \sqrt{x+4}} dx$</p> $I = \int_0^5 \frac{\sqrt{9-x}}{\sqrt{9-x} + \sqrt{x+4}} dx \quad \left \begin{array}{l} \text{Replace } x \rightarrow 5-x \\ \therefore 9-x \rightarrow x+4 \\ \& x+4 \rightarrow 9-x \end{array} \right.$ $\therefore I = \int_0^5 \frac{\sqrt{x+4}}{\sqrt{x+4} + \sqrt{9-x}} dx$ $\therefore 2I = \int_0^5 \frac{\sqrt{9-x} + \sqrt{x+4}}{\sqrt{9-x} + \sqrt{x+4}} dx$ $\therefore 2I = \int_0^5 1 \cdot dx$ $\therefore 2I = [x]_0^5$ $\therefore 2I = 5 - 0$ $\therefore I = \frac{5}{2}$ <p style="text-align: center;">OR</p> $\begin{aligned} I &= \int_0^5 \frac{\sqrt{9-x}}{\sqrt{9-x} + \sqrt{x+4}} dx \\ &= \int_0^5 \frac{\sqrt{9-(5-x)}}{\sqrt{9-(5-x)} + \sqrt{5-x+4}} dx \end{aligned}$ | 1 1 1 1 1 1 | 4 |
| | | | 1/2 1/2 1/2 1/2 | |



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|----------|-----------|---|--------------------------------|-------------|
| 3) | | $\therefore I = \int_0^5 \frac{\sqrt{x+4}}{\sqrt{x+4} + \sqrt{9-x}} dx$ $\therefore 2I = \int_0^5 \frac{\sqrt{9-x} + \sqrt{x+4}}{\sqrt{9-x} + \sqrt{x+4}} dx$ $\therefore 2I = \int_0^5 1 \cdot dx$ $\therefore 2I = [x]_0^5$ $\therefore 2I = 5 - 0$ $\therefore I = \frac{5}{2}$ | 1/2 1 1/2 1/2 1/2 | 4 |
| c) | | Find the area of region included between the parabola $y = x^2 + 1$ and the line $y = 2x + 1$. | | |
| Ans. | | $y = x^2 + 1 \text{ and } y = 2x + 1$ $\therefore x^2 + 1 = 2x + 1 \quad \text{or} \quad x^2 - 2x = 0$ $\therefore x(x-2) = 0$ $\therefore x = 0, 2$ $\therefore A = \int_0^a (y_2 - y_1) dx$ $= \int_0^2 [(2x+1) - (x^2 + 1)] dx$ $= \int_0^2 [2x - x^2] dx$ $= \left[x^2 - \frac{x^3}{3} \right]_0^2$ $= \left[2^2 - \frac{2^3}{3} \right] - [0 - 0]$ $= \frac{4}{3} \text{ or } 1.333$ | 1 1 1 1 1/2 1/2 | 4 |
| | | OR | | |
| | | $y = x^2 + 1 \text{ and } y = 2x + 1$ $\therefore x^2 + 1 = 2x + 1 \quad \text{or} \quad x^2 - 2x = 0$ $\therefore x(x-2) = 0$ $\therefore x = 0, 2$ $\therefore A = \int_0^2 [(x^2 + 1) - (2x + 1)] dx$ $= \int_0^2 [x^2 - 2x] dx$ | 1 1 | |



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| 3) | | $= \left[\frac{x^3}{3} - x^2 \right]_0^2$ $= \left[\frac{2^3}{3} - 2^2 \right] - [0 - 0]$ $= -\frac{4}{3} \quad \text{or} \quad -1.333$ $\therefore A = \frac{4}{3} \quad \text{or} \quad 1.333$ | 1 1/2 1/2 | |
| d) | | Solve the D. E. $x(1+y^2)dx + y(1+x^2)dy = 0$ | | |
| Ans. | | $\therefore \frac{x}{1+x^2}dx + \frac{y}{1+y^2}dy = 0$ $\therefore \int \frac{x}{1+x^2}dx + \int \frac{y}{1+y^2}dy = c$ $\therefore \frac{1}{2} \int \frac{2x}{1+x^2}dx + \frac{1}{2} \int \frac{2y}{1+y^2}dy = c$ $\therefore \frac{1}{2} \log(1+x^2) + \frac{1}{2} \log(1+y^2) = c$ | 1 1 1 1 | |
| | | OR | | |
| | | $\therefore \frac{x}{1+x^2}dx + \frac{y}{1+y^2}dy = 0$ $\therefore \int \frac{x}{1+x^2}dx + \int \frac{y}{1+y^2}dy = c$ | 1 1 | |
| | | $\text{Put } 1+x^2 = u \quad \text{and} \quad 1+y^2 = v$ $\therefore 2x dx = du \quad 2y dy = dv$ | 1/2 | |
| | | $\therefore \frac{1}{2} \int \frac{1}{u} du + \frac{1}{2} \int \frac{1}{v} dv = c$ $\therefore \frac{1}{2} \log(u) + \frac{1}{2} \log(v) = c$ $\therefore \frac{1}{2} \log(1+x^2) + \frac{1}{2} \log(1+y^2) = c$ | 1/2 1/2 1/2 | |
| | | OR | | |
| | | $M = x(1+y^2) \quad \therefore \frac{\partial M}{\partial y} = 2xy$ $N = y(1+x^2) \quad \therefore \frac{\partial N}{\partial x} = 2xy$ | 1 1 | |
| | | $\therefore \text{the equation is exact.}$ | | |
| | | $\int_{y \text{ constant}} M dx + \int_{\text{terms free from } x} N dy = c$ | | |



| Que. No. | Sub. Que. | Model answers | Marks | Total Marks |
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| 3) | | $\int x(1+y^2)dx + \int ydy = c$ $\therefore (1+y^2) \cdot \frac{x^2}{2} + \frac{y^2}{2} = c$ | 1 1 | 4 |
| e) | | Solve the D. E. $(4x+y)^2 \frac{dy}{dx} = 1$ | | |
| Ans. | | $Put \quad 4x+y = v$ $\therefore 4 + \frac{dy}{dx} = \frac{dv}{dx}$ $\therefore v^2 \left(\frac{dv}{dx} - 4 \right) = 1$ $\therefore \frac{dv}{dx} - 4 = \frac{1}{v^2}$ $\therefore \frac{dv}{dx} = \frac{1}{v^2} + 4 = \frac{1+4v^2}{v^2}$ $\therefore \frac{v^2}{1+4v^2} dv = dx$ $\therefore \int \frac{v^2}{1+4v^2} dv = \int dx$ $\therefore \frac{1}{4} \int \frac{4v^2}{1+4v^2} dv = \int dx$ $\therefore \frac{1}{4} \int \left[1 - \frac{1}{1+4v^2} \right] dv = \int dx$ $\therefore \frac{1}{4} \left[v - \frac{1}{2} \tan^{-1}(2v) \right] = x + c$ $\therefore \frac{1}{4} \left[4x + y - \frac{1}{2} \tan^{-1} 2(4x+y) \right] = x + c$ | $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ | 4 |
| f) | | Solve the D. E. $x(x+y)dy - y^2dx = 0$ | | |
| Ans. | | $x(x+y)dy - y^2dx = 0$ $\therefore x(x+y)dy = y^2dx$ $\therefore \frac{dy}{dx} = \frac{y^2}{x(x+y)} = \frac{y^2}{x^2 + xy}$ $Put \quad \frac{y}{x} = v \quad or \quad y = vx$ $\therefore \frac{dy}{dx} = v + x \frac{dv}{dx}$ $\therefore v + x \frac{dv}{dx} = \frac{(vx)^2}{x^2 + x.vx}$ | 1 | |



| Que. No. | Sub. Que. | Model answers | Marks | Total Marks |
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| 3) | | $\therefore v + x \frac{dv}{dx} = \frac{v^2}{1+v}$ $\therefore x \frac{dv}{dx} = \frac{v^2}{1+v} - v$ $\therefore x \frac{dv}{dx} = -\frac{v}{1+v}$ $\therefore \frac{1+v}{v} dv = -\frac{dx}{x}$ $\therefore \int \frac{1+v}{v} dv = -\int \frac{dx}{x}$ $\therefore \int \left[\frac{1}{v} + 1 \right] dv = -\int \frac{dx}{x}$ $\therefore \log v + v = -\log x + c$ $\therefore \log\left(\frac{y}{x}\right) + \frac{y}{x} = -\log x + c$ | $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$ | 4 |
| 4) | a) | <p>Attempt any FOUR of the following.</p> <p>Evaluate $\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\cot x}}$</p> $I = \int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\cot x}} = \int_{\pi/6}^{\pi/3} \frac{dx}{1 + \frac{\sqrt{\cos x}}{\sqrt{\sin x}}}$ $I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$ <p style="margin-left: 200px;"> $\boxed{\begin{array}{l} \text{Replace } x \rightarrow \pi/2 - x \\ \therefore \sin x \rightarrow \cos x \\ \& \cos x \rightarrow \sin x \end{array}}$ </p> $\therefore I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$ $\therefore 2I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x} + \sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$ $\therefore 2I = \int_{\pi/6}^{\pi/3} 1 \cdot dx$ $\therefore 2I = [x]_{\pi/6}^{\pi/3}$ $= \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$ $\therefore I = \frac{\pi}{12}$ | $\frac{1}{2}$ 1 1 $\frac{1}{2}$ $\frac{1}{2}$ | 4 |



| Que. No. | Sub. Que. | Model answers | Marks | Total Marks |
|----------|-----------|---|---|-------------|
| 4) | | <p style="text-align: center;">OR</p> $ \begin{aligned} I &= \int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\cot x}} \\ &= \int_{\pi/6}^{\pi/3} \frac{dx}{1 + \frac{\sqrt{\cos x}}{\sqrt{\sin x}}} \\ &= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx \\ &= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin\left(\frac{\pi}{2} - x\right)}}{\sqrt{\sin\left(\frac{\pi}{2} - x\right)} + \sqrt{\cos\left(\frac{\pi}{2} - x\right)}} dx \\ \therefore I &= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \\ \therefore 2I &= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x} + \sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \\ \therefore 2I &= \int_{\pi/6}^{\pi/3} 1 \cdot dx \\ \therefore 2I &= [x]_{\pi/6}^{\pi/3} \\ &= \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6} \\ \therefore I &= \frac{\pi}{12} \end{aligned} $ | 1/2 1/2 1 1/2 1/2 1/2 1/2 | 4 |
| b) | | Evaluate $\int_0^\pi \frac{x \sin x}{1 + \cos^2 x} dx$ | | |
| | Ans. | $ \begin{aligned} I &= \int_0^\pi \frac{x \sin x}{1 + \cos^2 x} dx \\ &= \int_0^\pi \frac{(\pi - x) \sin(\pi - x)}{1 + \cos^2(\pi - x)} dx \\ &= \int_0^\pi \frac{(\pi - x) \sin x}{1 + \cos^2 x} dx \\ &= \pi \int_0^\pi \frac{\sin x}{1 + \cos^2 x} dx - \int_0^\pi \frac{x \sin x}{1 + \cos^2 x} dx \\ &= \pi \int_0^\pi \frac{\sin x}{1 + \cos^2 x} dx - I \end{aligned} $ | 1/2 1/2 1/2 | |



| Que. No. | Sub. Que. | Model answers | Marks | Total Marks |
|----------|-----------|---|---------------------------------|-------------|
| 4) | | $\therefore 2I = \pi \int_0^\pi \frac{\sin x}{1 + \cos^2 x} dx$ <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> $\begin{aligned} &\text{Put } \cos x = t \\ &\therefore -\sin x dx = dt \\ &x \quad 0 \quad \pi \\ &t \quad 1 \quad -1 \end{aligned}$ </div> $\therefore 2I = -\pi \int_1^{-1} \frac{dt}{1+t^2}$ $\therefore 2I = -\pi \left[\tan^{-1} t \right]_1^{-1}$ $\therefore 2I = -\pi \left[\tan^{-1}(-1) - \tan^{-1}(1) \right]$ $\therefore 2I = \frac{\pi^2}{2}$ $\therefore I = \frac{\pi^2}{4}$ | 1/2 1/2 1/2 1/2 1/2 | |
| c) | | Find the area of the circle $x^2 + y^2 = 16$ by integration. | | |
| Ans. | | $x^2 + y^2 = 16$ $\therefore y = \sqrt{16 - x^2}$ $y = 0 \text{ gives } x^2 = 16$ $\therefore x = 4, -4$ $A = 4 \int_0^4 y dx$ $= 4 \int_0^4 \sqrt{16 - x^2} dx$ $= 4 \int_0^4 \sqrt{4^2 - x^2} dx$ $= 4 \left[\frac{x}{2} \sqrt{16 - x^2} + \frac{4^2}{2} \sin^{-1} \left(\frac{x}{4} \right) \right]_0^4$ $= 4 \left[\frac{4}{2} \sqrt{16 - 4^2} + \frac{4^2}{2} \sin^{-1}(1) \right] - 4 \left[0 + \frac{4^2}{2} \sin^{-1} 0 \right]$ $= 4 \left[0 + \frac{4^2}{2} \cdot \frac{\pi}{2} \right] - 4[0+0]$ $= 16\pi$ | 1 1 1 1 1 | 4 |
| | | Note: The above can also be solved on the same line as follows: $A = 2 \int_a^b y dx = 2 \int_{-4}^4 \sqrt{16 - x^2} dx = 16\pi$ OR $A = \int_a^b y dx = \int_{-4}^4 \sqrt{16 - x^2} dx = 4\pi \text{ and The Area} = 16\pi.$ | | |



| Que. No. | Sub. Que. | Model answers | Marks | Total Marks |
|----------|-----------|---|------------------|-------------|
| 4) | d) | Solve the D. E. $\frac{dy}{dx} + y \tan x = \cos^2 x$ | | |
| | Ans. | $\frac{dy}{dx} + y \tan x = \cos^2 x$ $\therefore P = \tan x \text{ and } Q = \cos^2 x$ $\therefore IF = e^{\int pdx} = e^{\int \tan x dx} = e^{\log \sec x} = \sec x$ $\therefore y \cdot IF = \int Q \cdot IF \cdot dx + c$ $\therefore y \cdot \sec x = \int \cos^2 x \cdot \sec x \cdot dx + c$ $\therefore y \cdot \sec x = \int \cos x \cdot dx + c$ $\therefore y \cdot \sec x = \sin x + c$ | 1 1 1 1 | 4 |
| | e) | Solve the D. E. $(e^x + 2xy^2 + y^3)dx + (a^y + 2x^2y + 3xy^2)dy = 0$ | | |
| | Ans. | $M = e^x + 2xy^2 + y^3$ $\therefore \frac{\partial M}{\partial y} = 4xy + 3y^2$ $N = a^y + 2x^2y + 3xy^2$ $\therefore \frac{\partial N}{\partial x} = 4xy + 3y^2$ <p><i>the equation is exact.</i></p> $\int_{y \text{ constant}} M dx + \int_{\text{terms free from } x} N dy = c$ $\int (e^x + 2xy^2 + y^3) dx + \int a^y dy = c$ $\therefore e^x + 2y^2 \cdot \frac{x^2}{2} + y^3 x + \frac{a^y}{\log a} = c$ $\text{or } e^x + x^2 y^2 + xy^3 + \frac{a^y}{\log a} = c$ | 1 1/2 1/2 | 4 |
| | f) | Verify that $y = \log x$ is a solution of $x \frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$ | | |
| | Ans. | $y = \log x$ $\therefore \frac{dy}{dx} = \frac{1}{x}$ $\therefore x \frac{dy}{dx} = 1$ $\therefore x \frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$ | 1 1 2 | 4 |



| Que. No. | Sub. Que. | Model answers | Marks | Total Marks |
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| 4) | | OR | | |
| | | $y = \log x$ $\therefore \frac{dy}{dx} = \frac{1}{x}$ $\therefore \frac{d^2y}{dx^2} = -\frac{1}{x^2}$ $\therefore x \frac{d^2y}{dx^2} + \frac{dy}{dx} = x \left(-\frac{1}{x^2} \right) + \frac{1}{x}$ $= -\frac{1}{x} + \frac{1}{x} = 0$ | 1 1 1 1 | 4 |
| 5) | | <hr/> Attempt any FOUR of the following. | | |
| | a) | A room has 3 electric lamps. From a collection of 15 electric bulbs of which only 10 are good, 3 are selected at random and put in the lamps. Find the probability that the room is lighted by at least one of the bulbs. | | |
| | Ans. | $Total = 10 \text{ Good} + 5 \text{ Bad} = 15$ $n = {}^{15} C_3$ $p = p(\text{room is lightened by at least one bulb})$ $= 1 - p(\text{room is not lightened})$ $= 1 - p(\text{bad bulbs})$ $= 1 - \frac{{}^5 C_3}{{}^{15} C_3}$ $= \frac{89}{91} \quad \text{or} \quad 0.978 \quad \dots (*)$ | 1 1 1 1 1 1 | 4 |
| | | OR | | |
| | | $Total = 10 \text{ Good} + 5 \text{ Bad} = 15$ $n = {}^{15} C_3$ $p = p(\text{room is lightened by at least one bulb})$ $= p(1G \& 2B \text{ or } 2G \& 1B \text{ or } 3G)$ $= \frac{{}^{10} C_1 \times {}^5 C_2 + {}^{10} C_2 \times {}^5 C_1 + {}^{10} C_3}{{}^{15} C_3}$ $= \frac{89}{91} \quad \text{or} \quad 0.978 \quad \dots (*)$ | 1 1 1 1 1 | 4 |
| | | Note: As the use of Non-programmable Electronic Pocket Calculator is permissible, calculating the value directly without using the actual formula of ${}^n C_r$ as shown in (*) is allowed and thus no mark to be deducted. | | |



| Que. No. | Sub. Que. | Model answers | Marks | Total Marks |
|----------|-----------|--|--|---------------|
| 5) | b) | If 20% of the bolts produced by a machine are defective. Find the probability that out of 4 bolts drawn i) One is defective. ii) At most two are defective. Ans. $p = \frac{20}{100} = 0.2$ $\therefore q = 0.8$ $\text{Here } n = 4$ i) $p = {}^n C_r p^r q^{n-r}$ $= {}^4 C_1 (0.2)^1 (0.8)^3$ $= 0.4096$ | $\frac{1}{2}$ | |
| | | ii) $p = p(0) + p(1) + p(2)$ $= {}^4 C_0 (0.2)^0 (0.8)^4 + {}^4 C_1 (0.2)^1 (0.8)^3 + {}^4 C_2 (0.2)^2 (0.8)^2$ $= 0.4096 + 0.4096 + 0.1536$ $= 0.9728$ | $\frac{1}{2}$ | |
| | | <i>OR</i> $p = 1 - p(\text{at least 3})$ $= 1 - [p(3) + p(4)]$ $= 1 - [{}^4 C_3 (0.2)^3 (0.8)^1 + {}^4 C_4 (0.2)^4 (0.8)^0]$ $= 0.9728$ | $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ | $\frac{1}{2}$ |
| | c) | <hr/> A box contains 10 red, 5 white, 5 black balls. Two balls are drawn at random. Find the probability that they are not of the same colour. Ans. $\text{Total Balls} = 10 + 5 + 5 = 20$ $n = n(S) = {}^{20} C_2 = 190$ $m = n(\text{not of same colour})$ $= n(1R1W \text{ or } 1W1B \text{ or } 1R1B)$ $= 10 \times 5 + 5 \times 5 + 10 \times 5$ $= 125$ $\therefore p = \frac{m}{n} = \frac{125}{190} \text{ or } 0.659$ | 1 1 1 1 | 4 |
| | | <i>OR</i> | | |



| Que. No. | Sub. Que. | Model answers | Marks | Total Marks |
|----------|-----------|---|-----------------------------|-------------|
| 5) | | $\begin{aligned} \text{Total Balls} &= 10 + 5 + 5 = 20 \\ n = n(S) &= {}^{20}C_2 = 190 \quad \dots(*) \\ p = p(\text{not of same colour}) &= p(1R1W \text{ or } 1W1B \text{ or } 1R1B) \\ &= \frac{{}^{10}C_1 \times {}^5C_1 + {}^5C_1 \times {}^5C_1 + {}^{10}C_1 \times {}^5C_1}{{}^{20}C_2} \quad \dots(**) \\ &= \frac{25}{38} \text{ or } 0.659 \end{aligned}$ <p>Note: If the step (*) is not written and directly the step (**) is written, full marks to be given.</p> <hr/> | 1 1 1 1 | 4 |
| d) | | $\begin{aligned} \text{Evaluate } \int \frac{dx}{4-5\cos x} \\ \text{Put } \tan \frac{x}{2} = t \\ \therefore dx = \frac{2dt}{1+t^2}, \quad \cos x = \frac{1-t^2}{1+t^2} \\ \int \frac{dx}{4-5\cos x} = \int \frac{\frac{2dt}{1+t^2}}{4-5\left(\frac{1-t^2}{1+t^2}\right)} \\ = \int \frac{\frac{2dt}{1+t^2}}{4(1+t^2)-5(1-t^2)} \\ = \int \frac{2dt}{4+4t^2-5+5t^2} \\ = 2 \int \frac{dt}{9t^2-1} \\ = 2 \int \frac{dt}{(3t)^2-1^2} \quad \text{or} \quad \frac{2}{9} \int \frac{dt}{t^2-\left(\frac{1}{3}\right)^2} \\ = 2 \cdot \frac{1}{2 \times 1} \log \left(\frac{3t-1}{3t+1} \right) \times \frac{1}{3} + c \quad \text{or} \quad \frac{2}{9} \cdot \frac{1}{2 \times \frac{1}{3}} \log \left(\frac{t-\frac{1}{3}}{t+\frac{1}{3}} \right) + c \\ = \frac{1}{3} \log \left(\frac{3 \tan \frac{x}{2} - 1}{3 \tan \frac{x}{2} + 1} \right) + c \end{aligned}$ | 1 1/2 1/2 1/2 1 | 4 |



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| 5) | e) | Evaluate $\int_0^{\pi/4} \log(1 + \tan x) dx$ | | |
| | Ans. | $I = \int_0^{\pi/4} \log(1 + \tan x) dx$ $= \int_0^{\pi/4} \log\left[1 + \tan\left(\frac{\pi}{4} - x\right)\right] dx$ $= \int_0^{\pi/4} \log\left[1 + \frac{1 - \tan x}{1 + \tan x}\right] dx$ $= \int_0^{\pi/4} \log\left[\frac{2}{1 + \tan x}\right] dx$ $= \int_0^{\pi/4} [\log 2 - \log(1 + \tan x)] dx$ $= \int_0^{\pi/4} \log 2 dx - \int_0^{\pi/4} \log(1 + \tan x) dx$ $= \int_0^{\pi/4} \log 2 dx - I$ $\therefore 2I = \int_0^{\pi/4} \log 2 dx$ $= \log 2 \int_0^{\pi/4} dx$ $= \log 2 [x]_0^{\pi/4}$ $= \frac{\pi}{4} \log 2$ $\therefore I = \frac{\pi}{8} \log 2$ | $\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ | 4 |
| | f) | Solve the D. E. $\frac{dy}{dx} + \frac{y}{x} = y^2$ | | |
| | Ans. | $\frac{dy}{dx} + \frac{y}{x} = y^2$ $\therefore \frac{1}{y^2} \frac{dy}{dx} + \frac{1}{x} \cdot \frac{1}{y} = 1$ $\text{Put } \frac{1}{y} = t$ $\therefore -\frac{1}{y^2} \frac{dy}{dx} = \frac{dt}{dx}$ $\therefore -\frac{dt}{dx} + \frac{1}{x} \cdot t = 1$ $\therefore \frac{dt}{dx} - \frac{1}{x} \cdot t = -1$ $P = -\frac{1}{x} \quad \text{and} \quad Q = -1$ | 1 $\frac{1}{2}$ | |





| Que. No. | Sub. Que. | Model answers | Marks | Total Marks |
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| 6) | c) | <p>In a sample of 1000 cases, the mean of certain test is 14 and standard deviation is 2.5. Assuming the distribution to be normal, find</p> <p>i) How many students score between 12 and 15? ii) How many students score above 18?</p> <p>Given $A(0.8) = 0.2881$, $A(0.4) = 0.1554$ and $A(1.6) = 0.4452$</p> | | |
| | Ans. | <p>Given $\bar{x} = 14$, $\sigma = 2.5$</p> $z = \frac{\bar{x} - x}{\sigma}$ <p>i) $z = \frac{12 - 14}{2.5} = -0.8$</p> $z = \frac{15 - 14}{2.5} = 0.4$ $\therefore p = A(-0.8 \leq z \leq 0.4) \quad \dots\dots\dots (*)$ $= A(-0.8 \leq z \leq 0) + A(0 \leq z \leq 0.4)$ $= A(0 \leq z \leq 0.8) + A(0 \leq z \leq 0.4)$ $= 0.2881 + 0.1554$ $= 0.4435$ $\therefore \text{No. of students} = 1000 \times 0.4435$ $= 443.5$ <p style="text-align: center;">\square 444</p> <p>ii) $z = \frac{18 - 14}{2.5} = 1.6$</p> $\therefore p = A(z \geq 1.6)$ $= A(0 \leq z) - A(0 \leq z \leq 1.6)$ $= 0.5 - A(0 \leq z \leq 1.6)$ $= 0.5 - 0.4452$ $= 0.0548$ $\therefore \text{No. of students} = 1000 \times 0.0548$ $= 54.8$ <p style="text-align: center;">\square 55</p> | <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> | |
| | | <p>(*) Note: As the area under Standard Normal Curve represents probability, many authors use symbol of probability p instead of the symbol of area A i. e., instead of writing $A(-0.8 \leq z \leq 0.4)$ it is also written as $p(-0.8 \leq z \leq 0.4)$.</p> | | |



| Que. No. | Sub. Que. | Model answers | Marks | Total Marks |
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| 6) | d) | A manufacturer can sell x items at price of Rs. $(330 - x)$ each. The cost of producing x items is Rs. $x^2 + 10x + 12$. How many items must be sold so that his profit is maximum. | | |
| | Ans. | Selling price = $x(330 - x)$ Cost price = $x^2 + 10x + 12$ But, profit = selling price – cost price $\therefore p = x(330 - x) - (x^2 + 10x + 12)$ $\therefore p = -2x^2 + 320x - 12$ $\therefore \frac{dp}{dx} = -4x + 320$ $\therefore \frac{d^2p}{dx^2} = -4$ For stationary values, $\frac{dp}{dx} = 0$ $\therefore -4x + 320 = 0$ $\therefore x = 80$ At $x = 80$, $\frac{d^2p}{dx^2} = -4 < 0$ \therefore At $x = 80$, p is maximum. Thus the manufacturer can sell maximum 80 items. | 1 $\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ | 4 |
| | e) | Find the equation of the tangents to the curve $y = x^2 - 2x - 3$, where it cuts x-axis. $y = x^2 - 2x - 3$ $\therefore \frac{dy}{dx} = 2x - 2$ The curve cuts x-axis, $\therefore y = 0$ $\therefore x^2 - 2x - 3 = 0$ $\therefore x = -1, 3$ \therefore the points on x-axis are $(-1, 0)$ and $(3, 0)$ \therefore the slope of tangent at $(-1, 0)$ is $m = \frac{dy}{dx} = 2(-1) - 2 = -4$ \therefore the equation of tangent is $y - 0 = -4(x + 1)$ $\therefore [y = -4x - 4] \text{ or } [4x + y + 4 = 0]$ | $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ | |



| Que. No. | Sub. Que. | Model answers | Marks | Total Marks |
|----------|-----------|--|-------|-------------|
| 6) | | <p>∴ the slope of tangent at (3, 0) is</p> $m = \frac{dy}{dx} = 2(3) - 2 = 4$ <p>∴ the equation of tangent is</p> $y - 0 = 4(x - 3)$ <p>∴ $y = 4x - 12$ or $4x - y - 12 = 0$</p> | 1/2 | |
| f) | | <p>Find the area of the region lying between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$.</p> | 1 | |
| Ans. | | <p>$y^2 = 4ax, x^2 = 4ay$</p> $\therefore \left(\frac{x^2}{4a}\right)^2 = 4ax \quad \left(\text{Also } y = 2\sqrt{a} \cdot \sqrt{x} \text{ and } y = \frac{x^2}{4a} \right)$ $\therefore x^4 = 64a^3x$ $\therefore x(x^3 - 64a^3) = 0$ $\therefore x = 0, x = 4a$ $A = \int_a^b (y_2 - y_1) dx$ $= \int_0^{4a} \left[2\sqrt{a} \cdot \sqrt{x} - \frac{x^2}{4a} \right] dx \quad \text{or also } \int_0^{4a} \left[\frac{x^2}{4a} - 2\sqrt{a} \cdot \sqrt{x} \right] dx$ $= \left[2\sqrt{a} \cdot \frac{2}{3}x^{3/2} - \frac{1}{4a} \cdot \frac{x^3}{3} \right]_0^{4a}$ $= \left[2\sqrt{a} \cdot \frac{2}{3}(4a)^{3/2} - \frac{1}{4a} \cdot \frac{(4a)^3}{3} \right] - 0$ $= \frac{16}{3}a^2$ | 1 | |
| | | | 1/2 | 4 |
| | | Important Note | | |
| | | <p>In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, FIRST SEE whether the method falls within the scope of the curriculum, and THEN ONLY give appropriate marks in accordance with the scheme of marking.</p> | | |