



Winter - 2013 Examination

Subject & Code: Applied Maths (17301)

Model Answer

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Que. No.	Sub. Que.	Model answers	Marks	Total Marks
		<p>Important Instructions to the Examiners:</p> <ol style="list-style-type: none">1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.3) The language errors such as grammatical, spelling errors should not be given more importance. (Not applicable for subject English and Communication Skills.)4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by the candidate and those in the model answer may vary. The examiner may give credit for any equivalent figure drawn.5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and the model answer.6) In case of some questions credit may be given by judgment on part of examiner of relevant answer based on candidate's understanding.7) For programming language papers, credit may be given to any other program based on equivalent concept.		



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1)	d)	<p>Evaluate $\int \frac{\cos(\log x)}{x} dx$</p> <p>Ans. $\int \frac{\cos(\log x)}{x} dx$</p> <p>$= \int \cos t dt$</p> <p>$= \sin t + c$</p> <p>$= \sin(\log x) + c$</p> <p style="text-align: center;">OR</p> <p>$I = \int \frac{\cos(\log x)}{x} dx$</p> <p>Put $\log x = t$</p> <p>$\therefore \frac{1}{x} dx = dt$</p> <p>$\therefore I = \int \cos t dt$</p> <p>$= \sin t + c$</p> <p>$= \sin(\log x) + c$</p> <hr style="border-top: 1px dashed black;"/>	<p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>	2
	e)	<p>Evaluate $\int \frac{dx}{x(x+1)}$</p> <p>Ans. $\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$</p> <p>$\therefore \boxed{1 = (x+1)A + xB}$</p> <p>Put $x = 0$</p> <p>$\therefore 1 = (0+1)A + 0$</p> <p>$\therefore \boxed{1 = A}$</p> <p>Put $x+1 = 0 \quad \therefore x = -1,$</p> <p>$\therefore 1 = 0 + (-1)B$</p> <p>$\therefore \boxed{-1 = B}$</p> <p>$\therefore \frac{1}{x(x+1)} = \frac{1}{x} + \frac{-1}{x+1}$</p> <p>$\int \frac{dx}{x(x+1)} = \int \left[\frac{1}{x} + \frac{-1}{x+1} \right] dx$</p> <p>$= \log x - \log(x+1) + c$</p> <p style="text-align: center;">OR</p>	<p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>	2



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1)		$\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$ $\therefore \boxed{1 = (x+1)A + xB}$ <p>For $x = 0$</p> $A = \frac{1}{x+1} = \frac{1}{0+1} = 1$ <p>For $x+1 = 0 \quad \therefore x = -1,$</p> $B = \frac{1}{x} = \frac{1}{-1} = -1$ $\therefore \frac{1}{x(x+1)} = \frac{1}{x} + \frac{-1}{x+1}$ $\int \frac{dx}{x(x+1)} = \int \left[\frac{1}{x} + \frac{-1}{x+1} \right] dx$ $= \log x - \log(x+1) + c$	<p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2} + \frac{1}{2}$</p>	2
	f)	<p>Evaluate $\int \tan^{-1} x dx$</p> <p>Ans. $\int \tan^{-1} x dx = \int 1 \cdot \tan^{-1} x dx$</p> $= \tan^{-1} x \int 1 dx - \int \left(\int 1 dx \right) \frac{d}{dx} (\tan^{-1} x) dx$ $= x \tan^{-1} x - \int x \cdot \frac{1}{x^2+1} \cdot dx$ $= x \tan^{-1} x - \frac{1}{2} \int \frac{2x}{x^2+1} \cdot dx$ $= x \tan^{-1} x - \frac{1}{2} \log(x^2+1) + c$	<p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>	
	g)	<p>Evaluate $\int_1^2 \frac{1}{3x-2} dx$</p> <p>Ans. $\int_1^2 \frac{1}{3x-2} dx = \left[\frac{\log(3x-2)}{3} \right]_1^2$</p> $= \frac{\log[3(2)-2]}{3} - \frac{\log[3(1)-2]}{3}$ $= \frac{\log 4}{3} - \frac{\log 1}{3}$ $= \frac{1}{3} \log 4$	<p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>	2



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1)	h)	<p>Find the area contained by the curve $y = 1 + x^3 + 2 \sin x$ from $x = 0$ to $x = \pi$.</p> $A = \int_a^b y dx$ $= \int_0^\pi (1 + x^3 + 2 \sin x) dx$ $= \left[x + \frac{x^4}{4} - 2 \cos x \right]_0^\pi$ $= \left[\pi + \frac{\pi^4}{4} - 2 \cos \pi \right] - [0 + 0 - 2 \cos 0]$ $= \pi + \frac{\pi^4}{4} + 4 \quad \text{or} \quad 31.494$	1/2 1/2 1/2 1/2	2
	i)	<p>Find the order and degree of the D. E. $\frac{d^2 y}{dx^2} = \sqrt{y - \frac{dy}{dx}}$.</p>		
	Ans.	<p>Order = 2.</p> $\frac{d^2 y}{dx^2} = \sqrt{y - \frac{dy}{dx}}$ $\therefore \left(\frac{d^2 y}{dx^2} \right)^2 = y - \frac{dy}{dx}$ <p>Degree = 2</p>	1 1/2 1/2	
	j)	<p>Form a differential equation if $y = A \sin x + B \cos x$.</p>		
	Ans.	$y = A \sin x + B \cos x$ $\therefore \frac{dy}{dx} = A \cos x - B \sin x$ $\therefore \frac{d^2 y}{dx^2} = -A \sin x - B \cos x$ $= -(A \sin x + B \cos x)$ $= -y$ $\therefore \frac{d^2 y}{dx^2} + y = 0$	1/2 1/2 1/2 1/2	
k)	<p>From a pack of 52 cards one is drawn at random. Find the probability of getting a king.</p>			
Ans.	$p = \frac{m}{n} = \frac{4}{52}$ $= \frac{1}{13} \quad \text{or} \quad 0.0769$	1 1/2 1/2	2	

OR



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2)		<p>∴ the equation of tangent is</p> $y + 2 = -\frac{31}{14}(x - 1)$ <p>∴ $14y + 28 = -31x + 31$</p> <p>∴ $\boxed{31x + 14y - 3 = 0}$ or $\boxed{31x + 14y = 3}$</p> <p>∴ the slope of normal $= -\frac{1}{m} = \frac{14}{31}$</p> <p>∴ the equation of tangent is</p> $y + 2 = \frac{14}{31}(x - 1)$ <p>∴ $31y + 62 = 14x - 14$</p> <p>∴ $\boxed{14x - 31y - 76 = 0}$ or $\boxed{-14x + 31y + 76 = 0}$</p>	<p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p>	4
	b)	<p>A beam is bent in the form of the curve $y = 2 \sin x - \sin 2x$. Find the radius of curvature of the beam at this point at $x = \frac{\pi}{2}$.</p>		
	Ans.	<p>$y = 2 \sin x - \sin 2x$</p> <p>∴ $\frac{dy}{dx} = 2 \cos x - 2 \cos 2x$</p> <p>& $\frac{d^2y}{dx^2} = -2 \sin x + 4 \sin 2x$</p> <p>∴ at $x = \frac{\pi}{2}$,</p> $\frac{dy}{dx} = 2 \cos\left(\frac{\pi}{2}\right) - 2 \cos 2\left(\frac{\pi}{2}\right) = 2$ <p>and $\frac{d^2y}{dx^2} = -2 \sin\left(\frac{\pi}{2}\right) + 4 \sin 2\left(\frac{\pi}{2}\right) = -2$</p> $\therefore \rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = \frac{\left[1 + (2)^2\right]^{\frac{3}{2}}}{-2} = -\frac{5^{3/2}}{2} \text{ or } -\frac{5\sqrt{5}}{2} \text{ or } -5.590$	<p>1/2</p> <p>1/2</p> <p>1</p> <p>1</p> <p>1</p>	4
	c)	<p>Find the maximum and minimum values of $x^3 - 18x^2 + 96x$.</p> <p>Let $y = x^3 - 18x^2 + 96x$</p> <p>∴ $\frac{dy}{dx} = 3x^2 - 36x + 96$</p> <p>∴ $\frac{d^2y}{dx^2} = 6x - 36$</p>	<p>1/2</p> <p>1/2</p>	



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2)		<p>For stationary values, $\frac{dy}{dx} = 0$</p> <p>$\therefore 3x^2 - 36x + 96 = 0$</p> <p>$\therefore x = 4, 8$</p> <p>At $x = 4$, $\frac{d^2y}{dx^2} = 6(4) - 36 = -12 < 0$</p> <p>$\therefore$ At $x = 4$, y has maximum value and it is</p> $y = (4)^3 - 18(4)^2 + 96(4) = 160$ <p>At $x = 8$, $\frac{d^2y}{dx^2} = 6(8) - 36 = 12 > 0$</p> <p>$\therefore$ At $x = 8$, y has minimum value and it is</p> $y = (8)^3 - 18(8)^2 + 96(8) = 128$	1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	4
	d)	Evaluate $\int \frac{1 - \tan x}{1 + \tan x} dx$		
	Ans.	$\int \frac{1 - \tan x}{1 + \tan x} dx = \int \tan\left(\frac{\pi}{4} - x\right) dx$ $= -\log \sec\left(\frac{\pi}{4} - x\right) + c$ <p style="text-align: center;">OR</p> $\int \frac{1 - \tan x}{1 + \tan x} dx = \int \frac{1 - \frac{\sin x}{\cos x}}{1 + \frac{\sin x}{\cos x}} dx$ $= \int \frac{\cos x - \sin x}{\cos x + \sin x} dx$ $= \log(\cos x + \sin x) + c$ <p style="text-align: center;">OR</p> $\int \frac{1 - \tan x}{1 + \tan x} dx = \int \frac{1 - \frac{\sin x}{\cos x}}{1 + \frac{\sin x}{\cos x}} dx$ $= \int \frac{\cos x - \sin x}{\cos x + \sin x} dx$	2 2	4
		$= \int \frac{1}{t} dx$ $= \log t + c$ $= \log(\cos x + \sin x) + c$	1+1 $\frac{1}{2}$ 1 $\frac{1}{2}$	4



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2)	e)	Evaluate $\int \sin^3 x dx$		
	Ans.	$\int \sin^3 x dx = \int \frac{1}{4}(3 \sin x - \sin 3x) dx$ $= \frac{1}{4} \left(-3 \cos x + \frac{\cos 3x}{3} \right) + c$ <p style="text-align: center;">OR</p> $\int \sin^3 x dx = \int \sin^2 x \cdot \sin x dx$ $= \int (1 - \cos^2 x) \sin x dx$ <div style="display: flex; align-items: center;"> <div style="flex: 1;"> $= -\int (1 - t^2) dt$ $= -\left(t - \frac{t^3}{3} \right) + c$ $= -\left(\cos x - \frac{\cos^3 x}{3} \right) + c$ </div> <div style="border-left: 1px solid black; padding-left: 5px; margin-left: 10px;"> $\begin{aligned} \text{Put } \cos x &= t \\ \therefore -\sin x dx &= dt \end{aligned}$ </div> </div>	2 2 1 1 1 1	4 4
	f)	Evaluate $\int \frac{x+1}{x(x^2-4)} dx$		
	Ans.	$I = \int \frac{x+1}{x(x^2-4)} dx = \int \frac{x+1}{x(x-2)(x+2)} dx$ $\frac{x+1}{x(x-2)(x+2)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2}$ <p>For $x=0$</p> $A = \frac{x+1}{(x-2)(x+2)} = \frac{0+1}{(0-2)(0+2)} = \boxed{-\frac{1}{4}}$ <p>For $x-2=0 \quad \therefore x=2,$</p> $B = \frac{x+1}{x(x+2)} = \frac{2+1}{2(2+2)} = \boxed{\frac{3}{8}}$ <p>For $x+2=0 \quad \therefore x=-2,$</p> $C = \frac{x+1}{x(x-2)} = \frac{-2+1}{-2(-2+2)} = \boxed{-\frac{1}{8}}$ $\therefore \frac{x+1}{x(x-2)(x+2)} = \frac{-1/4}{x} + \frac{3/8}{x-2} + \frac{-1/8}{x+2}$ $I = \int \left[\frac{-1/4}{x} + \frac{3/8}{x-2} + \frac{-1/8}{x+2} \right] dx$ $= -\frac{1}{4} \log x + \frac{3}{8} \log(x-2) - \frac{1}{8} \log(x+2) + c$ <p style="text-align: center;">OR</p>	1 1 1 1	4



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2)		<p style="text-align: center;">OR</p> $I = \int \frac{x+1}{x(x^2-4)} dx = \int \frac{x+1}{x(x-2)(x+2)} dx$ $\frac{x+1}{x(x-2)(x+2)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2}$ $\therefore \boxed{x+1 = (x-2)(x+2)A + x(x+2)B + x(x-2)C}$ <p>Put $x = 0$</p> $\therefore 0+1 = (0-2)(0+2)A + 0+0$ $\therefore \boxed{-\frac{1}{4} = A}$ <p>Put $x-2 = 0 \quad \therefore x = 2,$</p> $\therefore 2+1 = 0+2(2+2)B + 0$ $\therefore \boxed{\frac{3}{8} = B}$ <p>Put $x+2 = 0 \quad \therefore x = -2,$</p> $\therefore -2+1 = 0+0-2(-2-2)C$ $\therefore \boxed{-\frac{1}{8} = C}$ $\therefore \frac{x+1}{x(x-2)(x+2)} = \frac{-1/4}{x} + \frac{3/8}{x-2} + \frac{-1/8}{x+2}$ $I = \int \left[\frac{-1/4}{x} + \frac{3/8}{x-2} + \frac{-1/8}{x+2} \right] dx$ $= -\frac{1}{4} \log x + \frac{3}{8} \log(x-2) - \frac{1}{8} \log(x+2) + c$	1 1 1	4
3)	a)	<p>Attempt any FOUR of the following.</p> <p>Ans. Evaluate $\int_0^4 \frac{dx}{\sqrt{4x-x^2}}$</p> $4x-x^2 = 4-4+4x-x^2 = 4-(x-2)^2$ $= 2^2 - (x-2)^2$ $\int_0^4 \frac{dx}{\sqrt{4x-x^2}} = \int_0^4 \frac{dx}{\sqrt{2^2 - (x-2)^2}}$ $= \left[\sin^{-1} \left(\frac{x-2}{2} \right) \right]_0^4$ $= \sin^{-1} \left(\frac{4-2}{2} \right) - \sin^{-1} \left(\frac{0-2}{2} \right)$ $= \sin^{-1}(1) - \sin^{-1}(-1)$ $= \frac{\pi}{2} + \frac{\pi}{2} = \pi$	1 1 1	4



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3)		<p style="text-align: center;">OR</p> $\int_0^4 \frac{dx}{\sqrt{4x-x^2}} = \int_0^4 \frac{dx}{\sqrt{4-4+4x-x^2}}$ $= \int_0^4 \frac{dx}{\sqrt{4-(x-2)^2}}$ $= \int_0^4 \frac{dx}{\sqrt{2^2-(x-2)^2}}$ $= \left[\sin^{-1} \left(\frac{x-2}{2} \right) \right]_0^4$ $= \sin^{-1} \left(\frac{4-2}{2} \right) - \sin^{-1} \left(\frac{0-2}{2} \right)$ $= \sin^{-1} (1) - \sin^{-1} (-1)$ $= \frac{\pi}{2} + \frac{\pi}{2} = \pi$	1 1 1 1	4
	b)	<p>Evaluate $\int_0^5 \frac{\sqrt{9-x}}{\sqrt{9-x}+\sqrt{x+4}} dx$</p> $I = \int_0^5 \frac{\sqrt{9-x}}{\sqrt{9-x}+\sqrt{x+4}} dx$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-left: 20px;"> <p>Re place $x \rightarrow 5-x$ $\therefore 9-x \rightarrow x+4$ $\& x+4 \rightarrow 9-x$</p> </div> $\therefore I = \int_0^5 \frac{\sqrt{x+4}}{\sqrt{x+4}+\sqrt{9-x}} dx$ $\therefore 2I = \int_0^5 \frac{\sqrt{9-x}+\sqrt{x+4}}{\sqrt{9-x}+\sqrt{x+4}} dx$ $\therefore 2I = \int_0^5 1 \cdot dx$ $\therefore 2I = [x]_0^5$ $\therefore 2I = 5-0$ $\therefore I = \frac{5}{2}$ <p style="text-align: center;">OR</p> $I = \int_0^5 \frac{\sqrt{9-x}}{\sqrt{9-x}+\sqrt{x+4}} dx$ $= \int_0^5 \frac{\sqrt{9-(5-x)}}{\sqrt{9-(5-x)}+\sqrt{5-x+4}} dx$	1 1 1/2 1/2 1/2 1/2	



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3)		$\therefore I = \int_0^5 \frac{\sqrt{x+4}}{\sqrt{x+4} + \sqrt{9-x}} dx$ $\therefore 2I = \int_0^5 \frac{\sqrt{9-x} + \sqrt{x+4}}{\sqrt{9-x} + \sqrt{x+4}} dx$ $\therefore 2I = \int_0^5 1 \cdot dx$ $\therefore 2I = [x]_0^5$ $\therefore 2I = 5 - 0$ $\therefore I = \frac{5}{2}$	<p>1/2</p> <p>1</p> <p>1/2</p> <p>1/2</p> <p>1/2</p>	4
	c)	Find the area of region included between the parabola $y = x^2 + 1$ and the line $y = 2x + 1$.		
	Ans.	$y = x^2 + 1$ and $y = 2x + 1$ $\therefore x^2 + 1 = 2x + 1$ or $x^2 - 2x = 0$ $\therefore x(x - 2) = 0$ $\therefore x = 0, 2$ $\therefore A = \int_0^2 (y_2 - y_1) dx$ $= \int_0^2 [(2x + 1) - (x^2 + 1)] dx$ $= \int_0^2 [2x - x^2] dx$ $= \left[x^2 - \frac{x^3}{3} \right]_0^2$ $= \left[2^2 - \frac{2^3}{3} \right] - [0 - 0]$ $= \frac{4}{3}$ or 1.333	<p>1</p> <p>1</p> <p>1</p> <p>1/2</p> <p>1/2</p>	4
		OR		
		$y = x^2 + 1$ and $y = 2x + 1$ $\therefore x^2 + 1 = 2x + 1$ or $x^2 - 2x = 0$ $\therefore x(x - 2) = 0$ $\therefore x = 0, 2$ $\therefore A = \int_0^2 [(x^2 + 1) - (2x + 1)] dx$ $= \int_0^2 [x^2 - 2x] dx$	<p>1</p> <p>1</p>	



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3)		$= \left[\frac{x^3}{3} - x^2 \right]_0^2$ $= \left[\frac{2^3}{3} - 2^2 \right] - [0 - 0]$ $= -\frac{4}{3} \text{ or } -1.333$ $\therefore A = \frac{4}{3} \text{ or } 1.333$	1 1/2 1/2	4
	d)	Solve the D. E. $x(1+y^2)dx + y(1+x^2)dy = 0$		
	Ans.	$\therefore \frac{x}{1+x^2} dx + \frac{y}{1+y^2} dy = 0$ $\therefore \int \frac{x}{1+x^2} dx + \int \frac{y}{1+y^2} dy = c$ $\therefore \frac{1}{2} \int \frac{2x}{1+x^2} dx + \frac{1}{2} \int \frac{2y}{1+y^2} dy = c$ $\therefore \frac{1}{2} \log(1+x^2) + \frac{1}{2} \log(1+y^2) = c$ <p style="text-align: center;">OR</p> $\therefore \frac{x}{1+x^2} dx + \frac{y}{1+y^2} dy = 0$ $\therefore \int \frac{x}{1+x^2} dx + \int \frac{y}{1+y^2} dy = c$ <p>Put $1+x^2 = u$ and $1+y^2 = v$</p> $\therefore 2xdx = du \quad 2ydy = dv$ $\therefore \frac{1}{2} \int \frac{1}{u} du + \frac{1}{2} \int \frac{1}{v} dv = c$ $\therefore \frac{1}{2} \log(u) + \frac{1}{2} \log(v) = c$ $\therefore \frac{1}{2} \log(1+x^2) + \frac{1}{2} \log(1+y^2) = c$ <p style="text-align: center;">OR</p> $M = x(1+y^2) \quad \therefore \frac{\partial M}{\partial y} = 2xy$ $N = y(1+x^2) \quad \therefore \frac{\partial N}{\partial x} = 2xy$ <p>\therefore the equation is exact.</p> $\int_{y \text{ constant}} Mdx + \int \text{terms free from } x Ndy = c$	1 1 1 1 1 1 1/2 1/2 1/2 1 1	4



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3)		$\int x(1+y^2)dx + \int ydy = c$ $\therefore (1+y^2) \cdot \frac{x^2}{2} + \frac{y^2}{2} = c$	1 1	4
	e)	Solve the D. E. $(4x+y)^2 \frac{dy}{dx} = 1$ Ans. Put $4x+y = v$ $\therefore 4 + \frac{dy}{dx} = \frac{dv}{dx}$ $\therefore v^2 \left(\frac{dv}{dx} - 4 \right) = 1$ $\therefore \frac{dv}{dx} - 4 = \frac{1}{v^2}$ $\therefore \frac{dv}{dx} = \frac{1}{v^2} + 4 = \frac{1+4v^2}{v^2}$ $\therefore \frac{v^2}{1+4v^2} dv = dx$ $\therefore \int \frac{v^2}{1+4v^2} dv = \int dx$ $\therefore \frac{1}{4} \int \frac{4v^2}{1+4v^2} dv = \int dx$ $\therefore \frac{1}{4} \int \left[1 - \frac{1}{1+4v^2} \right] dv = \int dx$ $\therefore \frac{1}{4} \left[v - \frac{1}{2} \tan^{-1}(2v) \right] = x + c$ $\therefore \frac{1}{4} \left[4x + y - \frac{1}{2} \tan^{-1} 2(4x + y) \right] = x + c$	1/2 1/2 1/2	
	f)	Solve the D. E. $x(x+y)dy - y^2dx = 0$ Ans. $x(x+y)dy - y^2dx = 0$ $\therefore x(x+y)dy = y^2dx$ $\therefore \frac{dy}{dx} = \frac{y^2}{x(x+y)} = \frac{y^2}{x^2 + xy}$ Put $\frac{y}{x} = v$ or $y = vx$ $\therefore \frac{dy}{dx} = v + x \frac{dv}{dx}$ $\therefore v + x \frac{dv}{dx} = \frac{(vx)^2}{x^2 + x.vx}$	1	4



Que. No.	Sub. Que.	Model answers	Marks	Total Marks
3)		$\therefore v + x \frac{dv}{dx} = \frac{v^2}{1+v}$ $\therefore x \frac{dv}{dx} = \frac{v^2}{1+v} - v$ $\therefore x \frac{dv}{dx} = -\frac{v}{1+v}$ $\therefore \frac{1+v}{v} dv = -\frac{dx}{x}$ $\therefore \int \frac{1+v}{v} dv = -\int \frac{dx}{x}$ $\therefore \int \left[\frac{1}{v} + 1 \right] dv = -\int \frac{dx}{x}$ $\therefore \log v + v = -\log x + c$ $\therefore \log \left(\frac{y}{x} \right) + \frac{y}{x} = -\log x + c$	<p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1</p> <p>1/2</p>	4
4)	a)	<p>Attempt any FOUR of the following.</p> <p>Evaluate $\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\cot x}}$</p> $I = \int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\cot x}} = \int_{\pi/6}^{\pi/3} \frac{dx}{1 + \frac{\sqrt{\cos x}}{\sqrt{\sin x}}}$ $I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-left: 20px;"> <p>Replace $x \rightarrow \pi/2 - x$ $\therefore \sin x \rightarrow \cos x$ & $\cos x \rightarrow \sin x$</p> </div> $\therefore I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$ $\therefore 2I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x} + \sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$ $\therefore 2I = \int_{\pi/6}^{\pi/3} 1 \cdot dx$ $\therefore 2I = [x]_{\pi/6}^{\pi/3}$ $= \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$ $\therefore I = \frac{\pi}{12}$	<p>1</p> <p>1</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p>	



Que. No.	Sub. Que.	Model answers	Marks	Total Marks
4)		OR		
		$I = \int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\cot x}}$ $= \int_{\pi/6}^{\pi/3} \frac{dx}{1 + \frac{\sqrt{\cos x}}{\sqrt{\sin x}}}$ $= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$ $= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin\left(\frac{\pi}{2} - x\right)}}{\sqrt{\sin\left(\frac{\pi}{2} - x\right)} + \sqrt{\cos\left(\frac{\pi}{2} - x\right)}} dx$ $\therefore I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$ $\therefore 2I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x} + \sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$ $\therefore 2I = \int_{\pi/6}^{\pi/3} 1 \cdot dx$ $\therefore 2I = [x]_{\pi/6}^{\pi/3}$ $= \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$ $\therefore I = \frac{\pi}{12}$	1/2 1/2 1 1/2 1/2 1/2 1/2	4
	b)	Evaluate $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$		
	Ans.	$I = \int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$ $= \int_0^{\pi} \frac{(\pi - x) \sin(\pi - x)}{1 + \cos^2(\pi - x)} dx$ $= \int_0^{\pi} \frac{(\pi - x) \sin x}{1 + \cos^2 x} dx$ $= \pi \int_0^{\pi} \frac{\sin x}{1 + \cos^2 x} dx - \int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$ $= \pi \int_0^{\pi} \frac{\sin x}{1 + \cos^2 x} dx - I$	1/2 1/2	



Que. No.	Sub. Que.	Model answers	Marks	Total Marks
4)	d)	Solve the D. E. $\frac{dy}{dx} + y \tan x = \cos^2 x$		
	Ans.	$\frac{dy}{dx} + y \tan x = \cos^2 x$ $\therefore P = \tan x \text{ and } Q = \cos^2 x$ $\therefore IF = e^{\int p dx} = e^{\int \tan x dx} = e^{\log \sec x} = \sec x$ $\therefore y \cdot IF = \int Q \cdot IF \cdot dx + c$ $\therefore y \cdot \sec x = \int \cos^2 x \cdot \sec x \cdot dx + c$ $\therefore y \cdot \sec x = \int \cos x \cdot dx + c$ $\therefore y \cdot \sec x = \sin x + c$	1 1 1 1	4
	e)	Solve the D. E. $(e^x + 2xy^2 + y^3)dx + (a^y + 2x^2y + 3xy^2)dy = 0$		
Ans.	$M = e^x + 2xy^2 + y^3$ $\therefore \frac{\partial M}{\partial y} = 4xy + 3y^2$ $N = a^y + 2x^2y + 3xy^2$ $\therefore \frac{\partial N}{\partial x} = 4xy + 3y^2$ $\therefore \text{the equation is exact.}$ $\int_{y \text{ constant}} M dx + \int_{\text{terms free from } x} N dy = c$ $\int (e^x + 2xy^2 + y^3) dx + \int a^y dy = c$ $\therefore e^x + 2y^2 \cdot \frac{x^2}{2} + y^3 x + \frac{a^y}{\log a} = c$ $\text{or } e^x + x^2 y^2 + xy^3 + \frac{a^y}{\log a} = c$	1 1/2 1/2 1 1	4	
f)	Verify that $y = \log x$ is a solution of $x \frac{d^2 y}{dx^2} + \frac{dy}{dx} = 0$			
Ans.	$y = \log x$ $\therefore \frac{dy}{dx} = \frac{1}{x}$ $\therefore x \frac{dy}{dx} = 1$ $\therefore x \frac{d^2 y}{dx^2} + \frac{dy}{dx} = 0$	1 1 2		4



Que. No.	Sub. Que.	Model answers	Marks	Total Marks
4)		<p style="text-align: center;">OR</p> $y = \log x$ $\therefore \frac{dy}{dx} = \frac{1}{x}$ $\therefore \frac{d^2y}{dx^2} = -\frac{1}{x^2}$ $\therefore x \frac{d^2y}{dx^2} + \frac{dy}{dx} = x \left(-\frac{1}{x^2} \right) + \frac{1}{x}$ $= -\frac{1}{x} + \frac{1}{x} = 0$	1 1 1 1	4
5)	a)	<p>Attempt any FOUR of the following.</p> <p>A room has 3 electric lamps. From a collection of 15 electric bulbs of which only 10 are good, 3 are selected at random and put in the lamps. Find the probability that the room is lighted by at least one of the bulbs.</p> <p>Ans. $Total = 10 \text{ Good} + 5 \text{ Bad} = 15$ $n = {}^{15}C_3$ $p = p(\text{room is lightened by at least one bulb})$ $= 1 - p(\text{room is not lightened})$ $= 1 - p(\text{bad bulbs})$ $= 1 - \frac{{}^5C_3}{{}^{15}C_3}$ $= \frac{89}{91} \text{ or } 0.978 \dots (*)$</p> <p style="text-align: center;">OR</p> <p>$Total = 10 \text{ Good} + 5 \text{ Bad} = 15$ $n = {}^{15}C_3$ $p = p(\text{room is lightened by at least one bulb})$ $= p(1G \& 2B \text{ or } 2G \& 1B \text{ or } 3G)$ $= \frac{{}^{10}C_1 \times {}^5C_2 + {}^{10}C_2 \times {}^5C_1 + {}^{10}C_3}{{}^{15}C_3}$ $= \frac{89}{91} \text{ or } 0.978 \dots (*)$</p>	1 1 1 1 1 1	4 4
		<p>Note: As the use of Non-programmable Electronic Pocket Calculator is permissible, calculating the value directly without using the actual formula of nC_r as shown in (*) is allowed and thus no mark to be deducted.</p>		



Que. No.	Sub. Que.	Model answers	Marks	Total Marks
5)	b)	<p>If 20% of the bolts produced by a machine are defective. Find the probability that out of 4 bolts drawn</p> <p>i) One is defective. ii) At most two are defective.</p> <p>Ans. $p = \frac{20}{100} = 0.2$ $\therefore q = 0.8$ Here $n = 4$ i) $p = {}^n C_r p^r q^{n-r}$ $= {}^4 C_1 (0.2)^1 (0.8)^3$ $= 0.4096$</p> <p>ii) $p = p(0) + p(1) + p(2)$ $= {}^4 C_0 (0.2)^0 (0.8)^4 + {}^4 C_1 (0.2)^1 (0.8)^3 + {}^4 C_2 (0.2)^2 (0.8)^2$ $= 0.4096 + 0.4096 + 0.1536$ $= 0.9728$</p> <p>OR</p> <p>$p = 1 - p(\text{at least } 3)$ $= 1 - [p(3) + p(4)]$ $= 1 - [{}^4 C_3 (0.2)^3 (0.8)^1 + {}^4 C_4 (0.2)^4 (0.8)^0]$ $= 0.9728$</p>	<p>$\frac{1}{2}$</p> <p>1 $\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>	4
	c)	<p>A box contains 10 red, 5 white, 5 black balls. Two balls are drawn at random. Find the probability that they are not of the same colour.</p> <p>Ans. $Total\ Balls = 10 + 5 + 5 = 20$ $n = n(S) = {}^{20} C_2 = 190$ $m = n(\text{not of same colour})$ $= n(1R1W \text{ or } 1W1B \text{ or } 1R1B)$ $= 10 \times 5 + 5 \times 5 + 10 \times 5$ $= 125$ $\therefore p = \frac{m}{n} = \frac{125}{190} \text{ or } 0.659$</p> <p style="text-align: center;">OR</p>	<p>1</p> <p>1</p> <p>1</p>	4



Que. No.	Sub. Que.	Model answers	Marks	Total Marks
5)	e)	Evaluate $\int_0^{\pi/4} \log(1 + \tan x) dx$		
	Ans.	$I = \int_0^{\pi/4} \log(1 + \tan x) dx$ $= \int_0^{\pi/4} \log \left[1 + \tan \left(\frac{\pi}{4} - x \right) \right] dx$ $= \int_0^{\pi/4} \log \left[1 + \frac{1 - \tan x}{1 + \tan x} \right] dx$ $= \int_0^{\pi/4} \log \left[\frac{2}{1 + \tan x} \right] dx$ $= \int_0^{\pi/4} [\log 2 - \log(1 + \tan x)] dx$ $= \int_0^{\pi/4} \log 2 dx - \int_0^{\pi/4} \log(1 + \tan x) dx$ $= \int_0^{\pi/4} \log 2 dx - I$ $\therefore 2I = \int_0^{\pi/4} \log 2 dx$ $= \log 2 \int_0^{\pi/4} dx$ $= \log 2 [x]_0^{\pi/4}$ $= \frac{\pi}{4} \log 2$ $\therefore I = \frac{\pi}{8} \log 2$	<p>1/2</p> <p>1</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p>	4
	f)	Solve the D. E. $\frac{dy}{dx} + \frac{y}{x} = y^2$		
	Ans.	$\frac{dy}{dx} + \frac{y}{x} = y^2$ $\therefore \frac{1}{y^2} \frac{dy}{dx} + \frac{1}{x} \cdot \frac{1}{y} = 1$ <p>Put $\frac{1}{y} = t$</p> $\therefore -\frac{1}{y^2} \frac{dy}{dx} = \frac{dt}{dx}$ $\therefore -\frac{dt}{dx} + \frac{1}{x} \cdot t = 1$ $\therefore \frac{dt}{dx} - \frac{1}{x} \cdot t = -1$ <p>$P = -\frac{1}{x}$ and $Q = -1$</p>	<p>1</p> <p>1/2</p>	



Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)		<p>\therefore the slope of tangent at (3, 0) is</p> $m = \frac{dy}{dx} = 2(3) - 2 = 4$ <p>\therefore the equation of tangent is</p> $y - 0 = 4(x - 3)$ <p>\therefore $y = 4x - 12$ or $4x - y - 12 = 0$</p>	1/2	
	f)	<p>Find the area of the region lying between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$.</p>		
	Ans.	<p>$y^2 = 4ax, x^2 = 4ay$</p> <p>$\therefore \left(\frac{x^2}{4a}\right)^2 = 4ax \quad \left(\text{Also } y = 2\sqrt{a} \cdot \sqrt{x} \text{ and } y = \frac{x^2}{4a}\right)$</p> <p>$\therefore x^4 = 64a^3x$</p> <p>$\therefore x(x^3 - 64a^3) = 0$</p> <p>$\therefore x = 0, x = 4a$</p> <p>$A = \int_a^b (y_2 - y_1) dx$</p> <p>$= \int_0^{4a} \left[2\sqrt{a} \cdot \sqrt{x} - \frac{x^2}{4a} \right] dx \quad \text{or also } \int_0^{4a} \left[\frac{x^2}{4a} - 2\sqrt{a} \cdot \sqrt{x} \right] dx$</p> <p>$= \left[2\sqrt{a} \cdot \frac{2}{3} x^{3/2} - \frac{1}{4a} \cdot \frac{x^3}{3} \right]_0^{4a}$</p> <p>$= \left[2\sqrt{a} \cdot \frac{2}{3} (4a)^{3/2} - \frac{1}{4a} \cdot \frac{(4a)^3}{3} \right] - 0$</p> <p>$= \frac{16}{3} a^2$</p>	1 1 1/2 1/2	4
		<p style="text-align: center;">Important Note</p> <p style="text-align: center;">In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, FIRST SEE whether the method falls within the scope of the curriculum, and THEN ONLY give appropriate marks in accordance with the scheme of marking.</p>		